1. Bernoulli. Can we use probability models based on Bernoulli trials to investigate the following situations? Explain.
   a) We roll 50 dice to find the distribution of the number of spots on the faces.
   b) How likely is it that in a group of 120 the majority may have Type A blood, given that Type A is found in 43% of the population?
   c) We deal 5 cards from a deck and get all hearts. How likely is that?
   d) We wish to predict the outcome of a vote on the school budget, and poll 500 of the 3000 likely voters to see how many favor the proposed budget.
   e) A company realizes that about 10% of its packages are not being sealed properly. In a case of 24, is it likely that more than 3 are unsealed?

2. Bernoulli 2. Can we use probability models based on Bernoulli trials to investigate the following situations? Explain.
   a) You are rolling 5 dice and need to get at least two 6's to win the game.
   b) We record the eye colors found in a group of 500 people.
   c) A manufacturer recalls a doll because about 3% have buttons that are not properly attached. Customers return 37 of these dolls to the local toy store. Is the manufacturer likely to find any dangerous buttons?
   d) A city council of 11 Republicans and 8 Democrats picks a committee of 4 at random. What's the probability they choose all Democrats?
   e) A 2002 Rutgers University study found that 74% of high-school students have cheated on a test at least once. Your local high-school principal conducts a survey in homerooms and gets responses that admit to cheating from 322 of the 481 students.

3. Simulating the model. Think about the Tiger Woods picture search again. You are opening boxes of cereal one at a time looking for his picture, which is in 20% of the boxes. You want to know how many boxes you might have to open in order to find Tiger.
   a) Describe how you would simulate the search for Tiger using random numbers.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that you might find your first picture of Tiger in the first box, the second, etc.
   d) Calculate the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

4. Simulation II. You are one space short of winning a child's board game and must roll a 1 on a die to claim victory. You want to know how many rolls it might take.
   a) Describe how you would simulate rolling the die until you get a 1.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that you might win on the first roll, the second, the third, etc.
   d) Calculate the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

5. Tiger again. Let's take one last look at the Tiger Woods picture search. You know his picture is in 20% of the cereal boxes. You buy five boxes to see how many pictures of Tiger you might get.
   a) Describe how you would simulate the number of pictures of Tiger you might find in five boxes of cereal.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities that you get no pictures of Tiger, 1 picture, 2 pictures, etc.
   d) Calculate the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

6. Seatbelts. Suppose 75% of all drivers always wear their seatbelts. Let's investigate how many of the drivers might be belted among five cars waiting at a traffic light.
   a) Describe how you would simulate the number of seatbelt-wearing drivers among the five cars.
   b) Run at least 30 trials.
   c) Based on your simulation, estimate the probabilities there are no belted drivers, exactly one, two, etc.
   d) Calculate the actual probability model.
   e) Compare the distribution of outcomes in your simulation to the probability model.

7. Hoops. A basketball player has made 80% of his foul shots during the season. Assuming the shots are independent, find the probability that in tonight's game he
   a) misses for the first time on his fifth attempt.
   b) makes his first basket on his fourth shot.
   c) makes his first basket on one of his first 3 shots.

8. Chips. Suppose a computer chip manufacturer rejects 2% of the chips produced because they fail presale testing.
   a) What's the probability that the fifth chip you test is the first bad one you find?
   b) What's the probability you find a bad one within the first 10 you examine?

9. More hoops. For the basketball player in Exercise 7, what's the expected number of shots until he misses?

10. Chips ahoy. For the computer chips described in Exercise 8, how many do you expect to test before finding a bad one?

11. Blood. Only 4% of people have Type AB blood.
   a) On average, how many donors must be checked to find someone with Type AB blood?
   b) What's the probability that there is a Type AB donor among the first 5 people checked?
   c) What's the probability that the first Type AB donor will be found among the first 6 people?
   d) What's the probability that we won't find a Type AB donor before the 10th person?

12. Colorblindness. About 8% of males are colorblind. A researcher needs some colorblind subjects for an experiment and begins checking potential subjects.
   a) On average, how many men should the researcher expect to check to find one who is colorblind?
   b) What's the probability that she won't find anyone colorblind among the first 4 men she checks?
   c) What's the probability that the first colorblind man found will be the sixth person checked?
   d) What's the probability that she finds someone who is colorblind before checking the lOth man?

13. Lefties. Assume that 13% of people are left-handed. If we select 5 people at random, find the probability of each outcome described below.
   a) The first lefty is the fifth person chosen.
   b) There are some lefties among the 5 people.
   c) The first lefty is the second or third person.
   d) There are exactly 3 lefties in the group.
   e) There are at least 3lefties in the group.
   f) There are no more than 3 lefties in the group.

14. Arrows. An Olympic archer is able to hit the bull's-eye 80% of the time. Assume each shot is independent of the others. If she shoots 6 arrows, what's the probability of each result described below.
   a) Her first bull's-eye comes on the third arrow.
   b) She misses the bull's-eye at least once.
   c) Her first bull's-eye comes on the fourth or fifth arrow.
   d) She gets exactly 4 bull's-eyes.
   e) She gets at least 4 bull's-eyes.
   f) She gets at most 4 bull's-eyes.

15. Lefties redux. Consider our group of 5 people from Exercise 13.
   a) How many lefties do you expect?
   b) With what standard deviation?
   c) If we keep picking people until we find a lefty, how long do you expect it will take?

16. More arrows. Consider our archer from Exercise 14.
   a) How many bull's-eyes do you expect her to get?
   b) With what standard deviation?
   c) If she keeps shooting arrows until she hits the bull's eye, how long do you expect it will take?

17. Still more lefties. Suppose we choose 12 people instead of the 5 chosen in Exercise 13.
   a) Find the mean and standard deviation of the number of right-handers in the group.
   b) What's the probability that they're not all right handed?
   c) What's the probability that tl1ere are no more than 10 righties?
   d) What's the probability that there are exactly 6 of each?
   e) What's the probability that the majority is right handed?

18. Still more arrows. Suppose our archer from Exercise 14 shoots 10 arrows.
   a) Find the mean and standard deviation of the number of bull's-eyes she may get.
   b) What's the probability that she never misses?
   c) What's the probability that there are no more than 8 bull's-eyes?
   d) What's the probability that there are exactly 8 bull's eyes?
   e) What's the probability that she hits the bull's-eye more often than she misses?

19. Tennis, anyone? A certain tennis player makes a successful first serve 70% of the time. Assume that each serve is independent of the others. If she serves 6 times, what's the probability she gets
   a) all 6 serves in?
   b) exactly 4 serves in?
   c) at least 4 serves in?
   d) no more than 4 serves in?

20. Frogs. A wildlife biologist examines frogs for a genetic trait he suspects may be linked to sensitivity to industrial toxins in the environment. Previous research had established that this trait is usually found in 1 of every 8 frogs. He collects and examines a dozen frogs. If the frequency of the trait has not changed, what's the probability he finds the trait in
   a) none of the 12 frogs?
   b) at least 2 frogs?
   c) 3 or 4 frogs?
   d) no more than 4 frogs?

21. And more tennis. Suppose the tennis player in Exercise 19 serves 80 times in a match.
   a) What's the mean and standard deviation of the number of good first serves expected?
   b) Verify that you can use a Normal model to approximate the distribution of the number of good first serves.
   c) Use the 68-95-99.7 Rule to describe this distribution.
   d) What's the probability she makes at least 65 first serves?

22. More arrows. The archer in Exercise 14 will be shooting 200 arrows in a large competition.
   a) What are the mean and standard deviation of the number ofbull's-eyes she might get?
   b) Is a Normal model appropriate here? Explain.
   c) Use the 68-95-99.7 Rule to describe the distribution of the number of bull's-eyes she may get.
   d) Would you be surprised if she made only 140 bull's eyes? Explain.

23. Frogs, part II. Based on concerns raised by his preliminary research, the biologist in Exercise 20 decides to collect and examine 150 frogs .
   a) Assuming the frequency of the trait is still 1 in 8, determine the mean and standard deviation of the number of frogs with the trait he should expect to find in his sample.
   b) Verify that he can use a Normal model to approximate the distribution of the number of frogs with the trait.
   c) He found the trait in 22 of his frogs. Do you think this proves that the trait has become more common? Explain.

24. Apples. An orchard owner knows that he'll have to use about 6% of the apples he harvests for cider because they will have bruises or blemishes. He expects a tree to produce about 300 apples.
   a) Describe an appropriate model for the number of cider apples that may come from that tree. Justify your model.
   b) Find the probability there will be no more than a dozen cider apples.
   c) Is it likely there will be more than 50 cider apples? Explain.

25. Lefties again. A lecture hall has 200 seats with folding arm tablets, 30 of which are designed for left-handers. The average size of classes that meet there is 188, and we can assume that about 13% of students are left-handed. What's the probability that a right-handed student in one of these classes is forced to use a lefty arm tablet?

26. No-shows. An airline, believing that 5% of passengers fail to show up for flights, overbooks (sells more tickets than there are seats). Suppose a plane will hold 265 passengers, and the airline sells 275 tickets. What's the probability the airline will not have enough seats so someone gets bumped?

27. Annoying phone calls. A newly hired telemarketer is told he will probably make a sale on about 12% of his phone calls. The first week he called 200 people, but only made 10 sales. Should he suspect he was misled about the true success rate? Explain.

28. The euro. Shortly after the introduction of the euro coin in Belgium, newspapers around the world published articles claiming the coin is biased. The stories were based on reports that someone had spun the coin 250 times and gotten 140 heads- that's 56% heads. Do you think this is evidence that spinning a euro is unfair? Explain.

29. Seatbelts II. Police estimate that 80% of drivers now wear their seatbelts. They set up a safety roadblock, stopping cars to check for seatbelt use.
   a) How many cars do they expect to stop before finding a driver whose seatbelt is not buckled?
   b) What's the probability that the first unbelted driver is in the 6th car stopped?
   c) What's the probability that the first 10 drivers are all wearing their seatbelts?
   d) If they stop 30 cars during the first hour, find the mean and standard deviation of the number of drivers expected to be wearing seatbelts.
   e) If they stop 120 cars during this safety check, what's the probability they find at least 20 drivers not wearing their seatbelts?

30. Rickets. Vitamin D is essential for strong, healthy bones. Our bodies produce vitamin D naturally when sunlight falls upon the skin, or it can be taken as a dietary supplement. Although the bone disease rickets was largely eliminated in England during the 1950s, some people there are concerned that this generation of children is at increased risk because they are more likely to watch TV or play computer games than spend time outdoors. Recent research indicated that about 20% of British children are deficient in vitamin D. Suppose doctors test a group of elementary school children.
   a) What's the probability that the first vitamin D-deficient child is the 8th one tested?
   b) What's the probability that the first 10 children tested are all okay?
   c) How many kids do they expect to test before finding one who has this vitamin deficiency?
   d) They will test 50 students at the third grade level. Find the mean and standard deviation of the number who may be deficient in vitamin D.
   e) If they test 320 children at this school, what's the probability that no more than 50 of them have the vitamin deficiency?

31. ESP. Scientists wish to test the mind-reading ability of a person who claims to "have ESP." They use five cards with different and distinctive symbols (square, circle, triangle, line, squiggle). Someone picks a card at random and thinks about the symbol. The "mind reader" must correctly identify which symbol was on the card. If the test consists of 100 trials, how many would this person need to get right in order to convince you that ESP may actually exist? Explain.

32. True-False. A true-false test consists of 50 questions. How many does a student have to get right to convince you that he is not merely guessing? Explain.

33. Hot hand. A basketball player who ordinarily makes about 55% of his free throw shots has made 4 in a row. Is this evidence that he has a "hot hand" tonight? That is, is this streak so unusual that it means the probability he makes a shot must have changed? Explain.

34. New bow. Our archer in Exercise 14 purchases a new bow, hoping that it will improve her success rate to more than 80% bull's-eyes. She is delighted when she first tests her new bow and hits 6 consecutive bull's-eyes. Do you think this is compelling evidence that the new bow

Answers

1. a) These are not Bernoulli trials. The possible outcomes are 1, 2, 3, 4, 5, and 6. There are more than two possible outcomes.
   b) These may be considered Bernoulli trials. There are only two possible outcomes, Type A and not Type A. Assuming the 120 donors are representative of the population, the probability of having Type A blood is 43%. The trials are not independent, because the population is finite, but the 120 donors represent less than 10% of all possible donors.
   c) These are not Bernoulli trials. The probability of getting a heart changes as cards are dealt without replacement.
   d) These are not Bernoulli trials. We are sampling without replacement, so the trials are not independent. Samples without replacement may be considered Bernoulli trials if the sample size is less than 10% of the population, but 500 is more than 10% of 3000.
   e) These may be considered Bernoulli trials. There are only two possible outcomes, sealed properly and not sealed properly. The probability that a package is unsealed is constant, at about 10%, as long as the packages checked are a representative sample of all packages. Finally, the trials are not independent, since the total number of packages is finite, but the 24 packages checked probably represent less than 10% of the packages.
2. a) These may be considered Bernoulli trials. There are only two possible outcomes, getting a 6 and not getting a 6. The probability of getting a 6 is constant at 1/6. The rolls are independent of one another, since the outcome of one die roll doesn't affect the other rolls.
   b) These are not Bernoulli trials. There are more than two possible outcomes for eye color.
   c) These can be considered Bernoulli trials. There are only two possible outcomes, properly attached buttons and improperly attached buttons. As long as the button problem occurs randomly, the probability of a doll having improperly attached buttons is constant at about 3%. The trails are not independent, since the total number of dolls is finite, but 37 dolls is probably less than 10% of all dolls.
   d) These are not Bernoulli trials. The trials are not independent, since the probability of picking a council member with a particular political affiliation changes depending on who has already been picked. The 10% condition is not met, since the sample of size 4 is more than 10% of the population of 19 people.
   e) These may be considered Bernoulli trials. There are only two possible outcomes, cheating and not cheating. Assuming that cheating patterns in this school are similar to the patterns in the nation, the probability that a student has cheated is constant, at 74%. The trials are not independent, since the population of all students is finite, but 481 is less than 10% of all students.
3. a) Answers will vary. A component is the simulation of the picture in one box of cereal. One possible way to model this component is to generate random digits 0-9. Let 0 and 1 represent Tiger Woods and 2-9 a picture of another sports star. Each run will consist of generating random numbers until a 0 or 1 is generated. The response variable will be the number of digits generated until the first 0 or 1.
   b) Answers will vary.
   c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you get a picture of Tiger Woods in the first box. This is the number of trials in which a 0 or 1 was generated first, divided by the total number of trials. Perform similar calculations for the simulated probability that you have to wait until the second box, the third box, etc.
   d) Let X = the number of boxes opened until the first Tiger Woods picture is found.

X	1	2	3	4	5	6	7	8	≥ 9
P(X)	0.20	$(0.8)(0.2)$ $= 0.16$	$(0.8)^2(0.2)$ $= 0.128$	$(0.8)^3(0.2)$ $= 0.1024$	$(0.8)^4(0.2)$ $= 0.082$	$(0.8)^5(0.2)$ $= 0.066$	$(0.8)^6(0.2)$ $= 0.052$	$(0.8)^7(0.2)$ $= 0.042$	0.168

   e) Answers will vary.
4. a) Answers will vary. A component is the simulation of one die roll. One possible way to model this component is to generate random digits 1-6. Let 1 represent getting 1 (the roll you need and let 2-6 represent not getting the roll you need. Each run will consist of generating random numbers until 1 is generated. The response variable will be the number of digits generated until the first 1.
   b) Answers will vary.
   c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you roll a 1 on the first roll. This is the number of trials in which a 1 was generated first divided by the total number of trials. Perform similar calculations for the simulated probability that you have to wait until the second roll, the third roll, etc.
   d) Let X = the number of rolls until the first 1 is rolled.

X	1	2	3	4	5	6	7	8	≥ 9
P(X)	$\frac{1}{6}$	$\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$ $= 0.139$	$\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)$ $= 0.116$	$\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$ $= 0.096$	$\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$ $= 0.80$	$\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)$ $= 0.067$	$\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right)$ $= 0.056$	$\left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$ $= 0.047$	0.233

   e) Answers will vary.
5. a) Answers will vary. A component is the simulation of the picture in one box of cereal. One possible way to model this component is to generate random digits 0-9. Let 0 and 1 represent Tiger Woods and 2-9 a picture of another sports star. Each run will consist of generating five random numbers. The response variable will be the number of 0s and 1s in the five random numbers.
   b) Answers will vary.

c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you get no pictures of Tiger Woods in the five boxes. This is the number of trials in which neither 0 nor 1 were generated divided by the total number of trials. Perform similar calculations for the simulated probability that you would get one picture, 2 pictures, etc.

d) Let X = the number of Tiger Woods pictures in 5 boxes.

X	0	1	2	3	4	5
P(X)	$\binom{5}{0}(0.2)^0(0.8)^5$ $= 0.33$	$\binom{5}{1}(0.2)^1(0.8)^4$ $= 0.41$	$\binom{5}{2}(0.2)^2(0.8)^3$ $= 0.20$	$\binom{5}{3}(0.2)^3(0.8)^2$ $= 0.05$	$\binom{5}{4}(0.2)^4(0.8)^1$ $= 0.01$	$\binom{5}{5}(0.2)^5(0.8)^0$ < 0.0001

e) Answers will vary.

6. a) Answers will vary. A component is the simulation of one driver in a car. One possible way to model this component is to generate pairs of random digits 00-99. Let 01-75 represent a driver wearing his or her seatbelt and let 76-99 and 00 represent a driver not wearing his or her seatbelt. Each run will consist of generating five pairs of random digits. The response variable will be the number of pairs of digits that are 00-75.

b) Answers will vary.

c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that none of the five drivers are wearing seatbelts. This is the number of trials in which no pairs of digits were 00-75, divided by the total number of trials. Perform similar calculations for the simulated probability that one driver is wearing his or her seatbelt, two drivers, etc.

d) Let X = the number of drivers wearing seatbelts in 5 cars.

X	0	1	2	3	4	5
P(X)	$\binom{5}{0}(0.75)^0(0.25)^5$ $= 0.0001$	$\binom{5}{1}(0.75)^1(0.25)^4$ $= 0.01$	$\binom{5}{2}(0.75)^2(0.25)^3$ $= 0.09$	$\binom{5}{3}(0.75)^3(0.25)^2$ $= 0.26$	$\binom{5}{4}(0.75)^4(0.25)^1$ $= 0.40$	$\binom{5}{5}(0.75)^5(0.25)^0$ $= 0.24$

e) Answers will vary.

7. The player's shots may be considered Bernoulli trials. There are only two possible outcomes (make or miss), the probability of making a shot is constant (80%), and the shots are independent of one another (making, or missing, a shot does not affect the probability of making the next).

Let X = the number of shots until the first missed shot.

Let Y = the number of shots until the first made shot.

Since these problems deal with shooting until the first miss (or until the first made shot), a geometric model, either *Geom*(0.8) or *Geom*(0.2), is ap0propriate.

a) Use *Geom*(0.2). $P(X = 5) = (0.8)^4(0.2) = 0.08192$ (Four shots made, followed by a miss.)

b) Use *Geom*(0.8). $P(Y = 4) = (0.2)^3(0.8) = 0.0064$ (Three misses, then a made shot.)

c) Use *Geom*(0.8). $P(Y = 1) + P(Y = 2) + P(Y = 3) = (0.8)+(0.2)(0.8)+(0.2)^2(0.8) = 0.992$

8. The selection of chips may be considered Bernoulli trials. There are only two possible outcomes (fail testing and pass testing). Provided that the chips selected are a representative sample of all chips, the probability that a chip fails testing is constant at 2%. The trials are not independent, since the population of chips is finite, but we won't need to sample more than 10% of all chips. Let X = the number of chips required until the first bad chip. The appropriate model is *Geom*(0.0 2).

a) $P(X = 5) = (0.98)^4(0.02) = 0.0184$ (Four good chips, then a bad one.)

b) $P(1 \le X \le 10) = (0.02)+(0.98)(0.02)+(0.98)^2(0.02)+\ldots+(0.98)^9(0.02) = 0.183$

(Use the geometric model on a calculator or computer for this one!)

9. As determined in Exercise 3, the shots can be considered Bernoulli trials, and since the player is shooting until the first miss, *Geom*(0.2) is the appropriate model. $E(x) = \frac{1}{p} = \frac{1}{0.2} = 5$ shots. The player is expected to take 5 shots until the first miss.

10. As determined in Exercise 4, the selection of chips can be considered Bernoulli trials, and since the company is selecting until the first bad chip, *Geom*(0.0 2) is the appropriate model. $E(x) = \frac{1}{p} = \frac{1}{0.02} = 50$ chips. The first bad chip is expected to be the 50th chip selected.

11. These may be considered Bernoulli trials. There are only two possible outcomes, Type AB and not Type AB. Provided that the donors are representative of the population, the probability of having Type AB blood is constant at 4%. The trials are not independent, since the population is finite, but we are selecting fewer than 10% of all potential donors. Since we are selecting people until the first success, the model *Geom*(0.0 4) may be used. Let X = the number of donors until the first Type AB donor is found.

a) $E(x) = \frac{1}{p} = \frac{1}{0.04} = 25$ people. We expect the 25th person to be the first Type AB donor.

b) P(a Type AB donor among the first 5 people checked) = $(0.04)+(0.96)(0.04)+(0.96)^2(0.04)+...+(0.96)^4(0.04)=0.185$

c) P(a Type AB donor among the first 6 people checked) = $(0.04)+(0.96)(0.04)+(0.96)^2(0.04)+...+(0.96)^5(0.04)=0.217$

d) P(no Type AB donor before the 10th person checked) = $P(X > 9)$ = $(0.96)^9=0.693$. This one is a bit tricky. There is no implication that we actually find a donor on the 10th trial. We only care that nine trials passed with no Type AB donor.

12. These may be considered Bernoulli trials. There are only two possible outcomes, colorblind and not colorblind. As long as the men selected are representative of the population of all men, the probability of being colorblind is constant at about 8%. Trials are not independent, since the population is finite, but we won't be sampling more than 10% of the population.
Let X = the number of people checked until the first colorblind man is found. Since we are selecting people until the first success, the model *Geom*(0.08), may be used.

a) $E(x)=\frac{1}{p}=\frac{1}{0.08}=12.5$ people. We expect to examine 12.5 people until finding the first colorblind person.

b) P(no colorblind men among the first 4) = $P(X > 4)$ = $(0.92)^4=0.716$

c) P(first colorblind man is the sixth man checked) = $P(X = 6)$ = $(0.92)^5(0.08)=0.0527$

d) P(she finds a colorblind man before the tenth man) = $(0.08)+(0.92)(0.08)+(0.92)^2(0.08)+...+(0.92)^8(0.08)=0.528$

13. These may be considered Bernoulli trials. There are only two possible outcomes, lefthanded and not left-handed. Since people are selected at random, the probability of being left-handed is constant at about 13%. The trials are not independent, since the population is finite, but a sample of 5 people is certainly fewer than 10% of all people.
Let X = the number of people checked until the first lefty is discovered.
Let Y = the number of lefties among n = 5.

a) P(first lefty is the fifth person) = $P(X = 5)$ = $(0.87)^4(0.13)=0.0745$

b) P(some lefties among the 5 people) = 1 − P(no lefties among the first 5 people) = $1-\binom{5}{0}(0.13)^0(0.87)^5=0.502$

c) P(first lefty is second or third person) = $P(X = 2) + P(X = 3)$ = $(0.87)(0.13)+(0.87)^2(0.13)=0.211$

d) P(exactly 3 lefties in the group) = $\binom{5}{3}(0.13)^3(0.87)^2=0.0166$

e) P(at least 3 lefties in the group) = $\binom{5}{3}(0.13)^3(0.87)^2+\binom{5}{4}(0.13)^4(0.87)^1+\binom{5}{5}(0.13)^5(0.87)^0=0.0179$

f) P(at most 3 lefties in the group) = $\binom{5}{0}(0.13)^0(0.87)^5+...\binom{5}{3}(0.13)^3(0.87)^2=0.9987$

14. These may be considered Bernoulli trials. There are only two possible outcomes, hitting the bull's-eye and not hitting the bull's-eye. The probability of hitting the bull's-eye is given, p = 0.80. The shots are assumed to be independent.
Let X = the number of shots until the first bull's-eye.
Let Y = the number of bull's-eyes in n = 6 shots.

a) $(0.2)^2(0.8)=0.032$

b) $1-\binom{6}{6}(0.8)^6(0.2)^0=0.738$

c) $(0.2)^3(0.8)+(0.2)^4(0.8)=0.00768$

d) $\binom{6}{4}(0.8)^4(0.2)^2=0.246$

e) $\binom{6}{4}(0.8)^4(0.2)^2+\binom{6}{5}(0.8)^5(0.2)^1+\binom{6}{6}(0.8)^6(0.2)^0=0.901$

f) $\binom{6}{0}(0.8)^0(0.2)^6+...+\binom{6}{4}(0.8)^4(0.2)^2=0.345$

15. a) $E(Y) = np = 5(0.13) = 0.65$ lefties.
b) $SD(Y) = \sqrt{npq}=\sqrt{5(0.13)(0.87)}=0.75$ lefties

c) Let X = the number of people checked until the first lefty is discovered. $E(x)=\frac{1}{p}=\frac{1}{0.13}=7.69$ people

16. a) $E(Y) = np = 6(0.8) = 4.8$ bull's-eyes
b) $SD(Y) = \sqrt{npq}=\sqrt{6(0.8)(0.2)}=0.98$ bull's-eyes

c) Let X = the number of arrows shot until the first bull's-eye $E(x)=\frac{1}{p}=\frac{1}{0.8}=1.25$ shots

17. a) $E(Y) = np = 12(0.87) = 10.44$ righties. $SD(Y) = \sqrt{npq}=\sqrt{12(0.87)(0.13)}=1.16$ righties

b) $1-\binom{12}{12}(0.87)^{12}(0.13)^{0}=0.812$

c) $\binom{12}{0}(0.87)^{0}(0.13)^{12}+\ldots+\binom{12}{10}(0.87)^{10}(0.13)^{2}=0.475$

d) $\binom{12}{6}(0.87)^{6}(0.13)^{6}=0.00193$

e) $\binom{12}{7}(0.87)^{7}(0.13)^{5}+\ldots+\binom{12}{12}(0.87)^{12}(0.13)^{0}=0.998$

18. a) $E(Y) = np = 10(0\ .80) = 8$ bull's-eyes hit.. $SD(Y) = \sqrt{npq}=\sqrt{10(0.8)(0.2)}=1.26$ bull's-eyes hit.

b) $\binom{10}{10}(0.8)^{10}(0.2)^{0}=0.107$

c) $\binom{10}{0}(0.8)^{0}(0.2)^{10}+\ldots+\binom{10}{8}(0.8)^{8}(0.2)^{2}=0.624$

d) $\binom{10}{8}(0.8)^{8}(0.2)^{2}=0.302$

e) $\binom{10}{6}(0.8)^{6}(0.2)^{4}+\ldots+\binom{10}{10}(0.8)^{10}(0.2)^{0}=0.967$

19. a) $\binom{6}{6}(0.7)^{6}(0.3)^{0}=0.118$

b) $\binom{6}{4}(0.7)^{4}(0.3)^{2}=0.324$

c) $\binom{6}{4}(0.7)^{4}(0.3)^{2}+\ldots+\binom{6}{6}(0.7)^{6}(0.3)^{0}=0.744$

d) $\binom{6}{0}(0.7)^{0}(0.3)^{6}+\ldots+\binom{6}{4}(0.7)^{4}(0.3)^{2}=0.580$

20. a) $\binom{12}{0}(0.125)^{0}(0.875)^{12}=0.201$

b) $\binom{12}{2}(0.125)^{2}(0.875)^{10}+\ldots+\binom{12}{12}(0.125)^{12}(0.875)^{0}=0.453$

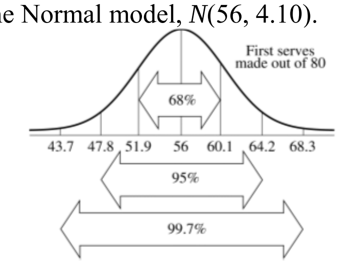
c) $\binom{12}{3}(0.125)^{3}(0.875)^{9}+\ldots+\binom{12}{4}(0.125)^{4}(0.875)^{8}=0.171$

d) $\binom{12}{0}(0.125)^{0}(0.875)^{12}+\ldots+\binom{12}{4}(0.125)^{4}(0.875)^{8}=0.989$

21. a) $E(X) = np = 80(0\ .70) = 56$ first serves in.. $SD(X) = \sqrt{npq}=\sqrt{80(0.7)(0.3)}=4.10$ first serves in.

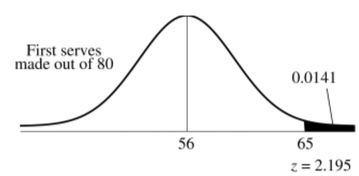
b) Since $np = 56$ and $nq = 24$ are both greater than 10, *Binom*(80,0 .70) may be approximated by the Normal model, *N*(56, 4.10).

c) According to the Normal model, in matches with 80 serves, she is expected to make between 51.9 and 60.1 first serves approximately 68% of the time, between 47.8 and 64.2 first serves approximately 95% of the time, and between 43.7 and 68.3 first serves approximately 99.7% of the time.



d) Using *Binom*(80, 0.70): $\binom{80}{65}(0.7)^{65}(0.3)^{15}+\ldots+\binom{80}{80}(0.7)^{80}(0.3)^{0}=0.0161$. According to the Binomial model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0161.

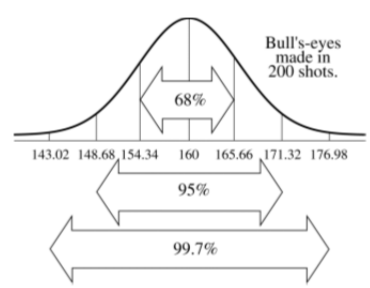
Using *N*(56, 4.10): $P(X \geq 65) \approx P\left(z > \frac{65-56}{4.1}\right) = P(z > 2.195) \approx$ 0.0 141. According to the Normal model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0141.



22. a) $E(Y) = np = 200(0.80) = 160$ bull's-eyes. $SD(Y) = \sqrt{npq}=\sqrt{200(0.8)(0.2)}=5.66$ bull's-eyes.

b) Since $np = 160$ and $nq = 40$ are both greater than 10, *Binom*(200,0.80) may be approximated by the Normal model, *N*(160, 5.66).

c) According to the Normal model, in matches with 200 arrows, she is expected to get between 154.34 and 165.66 bull's-eyes approximately 68% of the time, between 148.68 and 171.32 bull's-eyes approximately 95% of the time, and between 143.02 and 176.98 bull's-eyes approximately 99.7% of the time.

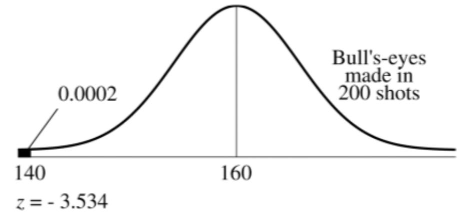


d) Using *Binom*(200, 0.80):

$\binom{200}{0}(0.8)^{0}(0.2)^{200}+\ldots+\binom{200}{140}(0.8)^{140}(0.2)^{60}=0.0005$. According to the Binomial model, the probability that she makes at most 140 bull's-eyes out of 200 is approximately 0.0005.

Using $N(160, 5.66)$: $P(Y \le 200) \approx P\left(z < \dfrac{140-160}{5.66}\right) = P(z < -3.534) \approx 0.0002$.

According to the Normal model, the probability that she hits at most 140 bull's-eyes out of 200 is approximately 0.0002. Using either model, it is apparent that it is very unlikely that the archer would hit only 140 bull's-eyes out of 200.



23. a) $E(X) = np = 150(0\ .125) = 18.75$ frogs. $SD(X) = \sqrt{npq} = \sqrt{150(0.125)(0.875)} = 4.05$ frogs.

b) Since $np = 18.75$ and $nq = 131.25$ are both greater than 10, $Binom(200,0.125)$ may be approximated by the Normal model, $N(18.75, 4.05)$.

c) Using $Binom(150, 0.125)$: $\binom{150}{22}(0.125)^{22}(0.875)^{128} + \ldots + \binom{150}{150}(0.125)^{150}(0.875)^{0} = 0.2433$. According to the Binomial model, the probability that at least 22 frogs out of 150 have the trait is approximately 0.2433.

Using $N(18.75, 4.05)$: $P(X \ge 22) \approx P\left(z > \dfrac{22-18.75}{4.05}\right) = P(z > 0.802) \approx 0.2111$. According to the Normal model, the probability that at least 22 frogs out of 150 have the trait is approximately 0.2111.

Using either model, the probability that the biologist discovers 22 of 150 frogs with the trait simply as a result of natural variability is quite high. This doesn't prove that the trait has become more common.

24. a) $E(X) = np = 300(0.06) = 18$ cider apples. $SD(X) = \sqrt{npq} = \sqrt{300(0.06)(0.94)} = 4.11$ cider apples. Since $np = 18$ and $nq = 282$ are both greater than 10, $Binom(300,0.06)$ may be approximated by the Normal model, $N(18, 4.11)$.

b) Using $Binom(300, 0.06)$: $\binom{300}{0}(0.06)^{0}(0.94)^{300} + \ldots + \binom{300}{12}(0.06)^{12}(0.94)^{288} = 0.085$. According to the Binomial model, the probability that no more than 12 cider apples come from the tree is approximately 0.085.

Using $N(18, 4.11)$: $P(X \le 12) \approx P\left(z < \dfrac{12-18}{4.11}\right) = P(z < -1.460)0 \approx .072$. According to the Normal model, the probability that no more than 12 apples out of 300 are cider apples is approximately 0.072.

c) It is extremely unlikely that the tree will bear more than 50 cider apples. Using the Normal model, $N(18, 4.11)$, 50 cider apples is approximately 7.8 standard deviations above the mean.

25. Using $Binom(188, 0.87)$: $\binom{188}{171}(0.87)^{171}(0.13)^{17} + \ldots + \binom{188}{188}(0.87)^{188}(0.13)^{0} = 0.061$. According to the binomial model, the probability that a right-handed student has to use a left-handed desk is approximately 0.061.

Using $N(163.56, 4.61)$: $E(X) = np = 188(0.87) = 163.56$ righties. $SD(X) = \sqrt{npq} = \sqrt{188(0.87)(0.13)} = 4.61$ righties.

Since $np = 163.56$ and $nq = 24.44$ are both greater than 10, $Binom(188,0.87\ )$ may be approximated by the Normal model, $N(163.56, 4.61)$. $P(X \ge 171) \approx P\left(z > \dfrac{171-163.56}{4.61}\right) = P(z > 1.614) \approx 0.0\ 53$ According to the Normal model, the probability that there are at least 171 righties in the class of 188 is approximately 0.0533.

26. Using $Binom(275, 0.95)$: $\binom{275}{266}(0.95)^{266}(0.05)^{9} + \ldots + \binom{275}{266}(0.95)^{266}(0.05)^{0} = 0.116$. According to the binomial model, the probability someone on the flight must be bumped is approximately 0.116.

Using $N(261.25, 3.61)$: $E(X) = np = 275(0.95\ ) = 261.25$ passengers. $SD(X) = \sqrt{npq} = \sqrt{275(0.95)(0.05)} = 3.61$ passengers.

Since $np = 261.25$ and $nq = 13.75$ are both greater than 10, $Binom(275,0.95\ )$ may be approximated by the Normal model, $N(261.25, 3.61)$. $P(X \ge 266) \approx P\left(z > \dfrac{266-261.25}{3.61}\right) = P(z > 1.316) \approx 0.0941$. According to the Normal model, the probability that at least 266 passengers show up is approximately 0.0941.

27. Using $Binom(200, 0.12)$: $\binom{200}{0}(0.12)^{0}(0.88)^{200} + \ldots + \binom{200}{10}(0.12)^{10}(0.88)^{190} = 0.0006$. According to the Binomial model, the probability that the telemarketer would make at most 10 sales is approximately 0.0006.

Using $N(24, 4.60)$: $E(X) = np = 200(0.12) = 24$ sales. $SD(X) = \sqrt{npq} = \sqrt{200(0.12)(0.88)} = 4.60$ sales.

Since $np = 24$ and $nq = 176$ are both greater than 10, $Binom(200,0.12)$ may be approximated by the Normal model, $N(24, 4.60)$.

$P(X \le 10) \approx P\left(z < \dfrac{10-24}{4.60}\right) = P(z < -3.043) \approx 0.0012$. According to the Normal model, the probability that the telemarketer would make at most 10 sales is approximately 0.0012.

Since the probability that the telemarketer made 10 sales, given that the 12% of calls result in sales is so low, it is likely that he was misled about the true success rate.

28. Using *Binom*(250, 0.5): $\binom{250}{140}(0.5)^{140}(0.5)^{110}+\ldots+\binom{250}{250}(0.5)^{250}(0.5)^{0}=0.0332$. According to the Binomial model, the probability that a fair Belgian euro comes up heads at least 140 times is 0.0332.

Using $N(125, 7.91)$: $E(X) = np = 250(0.05) = 125$ heads. $SD(X) = \sqrt{npq}=\sqrt{250(0.5)(0.5)}=7.91$ heads.

Since $np = 125$ and $nq = 125$ are both greater than 10, *Binom*(250, 0.5) may be approximated by the Normal model, $N(125, 7.91)$.

$P(X \geq 140)1\approx P\left(z>\frac{140-125}{7.91}\right) = P(z > 1.896) \approx 0.0290$. According to the Normal model, the probability that a fair Belgian euro lands heads at least 140 out of 250 spins is approximately 0.0290.

Since the probability that a fair Belgian euro lands heads at least 140 out of 250 spins is low, it is unlikely that the euro spins fairly. However, the probability is not extremely low, and we aren't sure of the source of the data, so it might be a good idea to spin it some more.

29. a) Let X = the number of cars stopped before finding a driver whose seat belt is not buckled. Use *Geom*(0.2) to model the situation. $E(X)=\frac{1}{p}=\frac{1}{0.2}=5$ cars.

b) P(First unbelted driver is in the sixth car) = $P(X = 6)$ = $(0.8)^{5}(0.2) \approx 0.066$

c) P(The first ten drivers are wearing seatbelts) = $(0.8)^{10} \approx 0.107$

d) Let Y = the number of drivers wearing their seatbelts in 30 cars. Use *Binom*(30, 0.8). $E(Y) = np = 30(0.8) = 24$ drivers. $SD(Y) = \sqrt{npq}=\sqrt{30(0.8)(0.2)}=2.19$ drivers.

e) Let W = the number of drivers not wearing their seatbelts in 120 cars.

Using *Binom*(120, 0.2): $\binom{120}{20}(0.2)^{20}(0.8)^{100}+\ldots+\binom{120}{120}(0.2)^{120}(0.8)^{0}=0.848$. According to the Binomial model, the probability that at least 20 out of 120 drivers are not wearing their seatbelts is approximately 0.848.

Using $N(24, 4.38)$: $E(W) = np = 120(0.2) = 24$ drivers. $SD(W) = \sqrt{npq}=\sqrt{120(0.2)(0.8)}=4.38$ drivers.

Since $np = 24$ and $nq = 96$ are both greater than 10, *Binom*(120,0.2) may be approximated by the Normal model, $N(24, 4.38)$.

$P(W \geq 20) \approx P\left(z>\frac{20-24}{4.38}\right) = P(z > -0.913) \approx 0.8194$. According to the Normal model, the probability that at least 20 out of 120 drivers stopped are not wearing their seatbelts is approximately 0.8194.

30. a) Let X = the number of students tested before finding a student who is vitamin D deficient. Use *Geom*(0.2) to model the situation. P(First vitamin D deficient child is the eighth one tested) = $P(X = 8)$ = $(0.8)^{7}(0.2) \approx 0.042$

b) P(The first ten children tested are okay) = $(0.8)^{10} \approx 0.107$

c) $E(X)=\frac{1}{p}=\frac{1}{0.2}=5$ kids.

d) Let Y = the number of children who are vitamin D deficient out of 50 children.

Use *Binom*(50, 0.2). $E(Y) = np = 50(0.2) = 10$ children. $SD(Y) = \sqrt{npq}=\sqrt{50(0.2)(0.8)}=2.83$ children.

e) Using *Binom*(320, 0.2): $\binom{320}{0}(0.2)^{0}(0.8)^{320}+\ldots+\binom{320}{50}(0.2)^{50}(0.8)^{270}=0.027$. According to the Binomial model, the probability that no more than 50 of the 320 children have the vitamin D deficiency is approximately 0.027.

Using $N(64, 7.16)$: $E(Y) = np = 320(0.2) = 64$ children. $SD(Y) = \sqrt{npq}=\sqrt{320(0.2)(0.8)}=7.16 \approx 7.16$ children.

Since $np = 64$ and $nq = 256$ are both greater than 10, *Binom*(320,0.2) may be approximated by the Normal model, $N(64, 7.16)$.

$P(Y \leq 50) \approx P\left(z<\frac{50-64}{7.16}\right) = P(z < -1.955) \approx 0.0253$..According to the Normal model, the probability that no more than 50 out of 320 children have the vitamin D deficiency is approximately 0.0253.

31. $E(X) = np = 100(0.2) = 20$ correct identifications. $SD(X) = \sqrt{npq}=\sqrt{100(0.2)(0.8)}=4$ correct identifications. Answers may vary. In order be convincing, the "mind reader" would have to identify at least 32 out of 100 cards correctly, since 32 is three standard deviations above the mean. Identifying fewer cards than 32 could happen too often, simply due to chance.

32. $E(X) = np = 50(0.5) = 25$ correct answers. $SD(X) = \sqrt{npq}=\sqrt{50(0.5)(0.5)}=3.54$ correct answers. Answers may vary. In order be convincing, the student would have to answer at least 36 out of 50 questions correctly, since 36 is approximately three standard deviations above the mean. Answering fewer than 36 questions correctly could happen too often, simply due to chance.

33. A streak like this is not unusual. The probability that he makes 4 in a row with a 55% free throw percentage is $(0.55)(0.55)(0.55)(0.55) \approx 0.09$. We can expect this to happen nearly one in ten times for every set of 4 shots that he makes. One out of ten times is not that unusual.
34. A streak like this is not unusual. The probability that she makes 6 consecutive bulls-eyes with an 80% bulls-eye percentage is $(0.8)(0.8)(0.8)(0.8)(0.8) \approx 0.26$. If she were to shoot several flights of 6 arrows, she is expected to get 6 bulls-eyes about 26% of the time. An event that happens due to chance about one out of four times is not that unusual.