Random Variables

1. Expected value. Find the expected value of each random variable:

a)

x	10	20	30
P(X=x)	0.3	0.5	0.2

b)

x	2	4	6	8
P(X=x)	0.3	0.4	0.2	0.1

2. Expected value. Find the expected value of each random variable:

a)

x	1	2	3
P(X=x)	0.2	0.4	0.4

b)

x	100	200	300	400
P(X=x)	0.1	0.2	0.5	0.2

3. Pick a card, any card. You draw a card from a deck. If you get a red card, you win nothing. If you get a spade, you win $5. For any club, you win $10 plus an extra $20 for the ace of clubs.
a) Create a probability model for the amount you win at this game.
b) Find the expected amount you'll win.
c) How much would you be willing to pay to play this game?

4. You bet! You roll a die. If it comes up a 6, you win $100. If not, you get to roll again. If you get a 6 the second time, you win $50. If not, you lose.
a) Create a probability model for the amount you win at this game.
b) Find the expected amount you'll win.
c) How much would you be willing to pay to play this game?

5. Kids. A couple plans to have children until they get a girl, but they agree that they will not have more than three children even if all are boys. (Assume boys and girls are equally likely.)
a) Create a probability model for the number of children they'll have.
b) Find the expected number of children.
c) Find the expected number of boys they'll have.

6. Carnival. A carnival game offers a $100 cash prize for anyone who can break a balloon by throwing a dart at it. It costs $5 to play, and you're willing to spend up to $20 trying to win. You estimate that you have about a 10% chance of hitting the balloon on any throw.
a) Create a probability model for this carnival game.
b) Find the expected number of darts you'll throw.
c) Find your expected winnings.

7. Software. A small software company bids on two contracts. It anticipates a profit of $50,000 if it gets the larger contract and a profit of $20,000 on the smaller contract. The company estimates there's a 30% chance it will get the larger contract and a 60% chance it will get the smaller contract. Assuming the contracts will be awarded independently, what's the expected profit?

8. Racehorse. A man buys a racehorse for $20,000, and enters it in two races. He plans to sell the horse afterward, hoping to make a profit. If the horse wins both races, its value will jump to $100,000. If it wins one of the races, it will be worth $50,000. If it loses both races, it will be worth only $10,000. The man believes there's a 20% chance that the horse will win the first race and a 30% chance it will win the second one. Assuming that the two races are independent events, find the man's expected profit.

9. Variation 1. Find the standard deviations of the random variables in Exercise 1.

10. Variation 2. Find the standard deviations of the random variables in Exercise 2.

11. Pick another card. Find the standard deviation of the amount you might win drawing a card in Exercise 3.

12. The die. Find the standard deviation of the amount you might win rolling a die in Exercise 4.

13. Kids. Find the standard deviation of the number of children the couple in Exercise 5 may have.

14. Darts. Find the standard deviation of your winnings throwing darts in Exercise 6.

15. Repairs. The probability model below describes the number of repair calls that an appliance repair shop may receive during an hour.

Repair Calls	0	1	2	3
Probability	0.1	0.3	0.4	0.2

a) How many calls should the shop expect per hour?
b) What is the standard deviation?

16. Red lights. A commuter must pass through five traffic lights on her way to work, and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits, as shown below.

X=#of red	0	1	2	3	4	5
P(X=x)	0.05	0.25	0.35	0.15	0.15	0.05

a) How many red lights should she expect to hit each day?
b) What's the standard deviation?

17. Defects. A consumer organization inspecting new cars found that many had appearance defects (dents, scratches, paint chips, etc.). While none had more than three of these defects, 7% had three, 11% two, and 21% one defect. Find the expected number of appearance defects in a new car, and the standard deviation.

18. Insurance. An insurance policy costs \$100, and will pay policyholders \$10,000 if they suffer a major injury (resulting in hospitalization) or \$3,000 if they suffer a minor injury (resulting in lost time from work). The company estimates that each year 1 in every 2,000 policyholders may have a major injury, and 1 in 500 a minor injury.
a) Create a probability model for the profit on a policy.
b) What's the company's expected profit on this policy?
c) What's the standard deviation?

19. Contest. You play two games against the same opponent. The probability you win the first game is 0.4. If you win the first game, the probability you also win the second is 0.2. If you lose the first game, the probability that you win the second is 0.3.
a) Are the two games independent? Explain your answer.
b) What's the probability you lose both games?
c) What's the probability you win both games?
d) Let random variable X be the number of games you win. Find the probability model for X.
e) What are the expected value and standard deviation of X?

20. Contracts. Your company bids for two contracts. You believe the probability you get contract #1 is 0.8. If you get contract #1, the probability you also get contract #2 will be 0.2, and if you do not get #1, the probability you get #2 will be 0.3.
a) Are the two contracts independent? Explain.
b) Find the probability you get both contracts.
c) Find the probability you get no contract.
d) Let X be the number of contracts you get. Find the probability model for X.
e) Find the expected value and standard deviation of X.

21. Batteries. In a group of 10 batteries, 3 are dead. You choose 2 batteries at random.
a) Create a probability model for the number of good batteries you get.
b) What's the expected number of good ones you get?
c) What's the standard deviation?

22. Kittens. In a litter of seven kittens, three are female. You pick two kittens at random.
a) Create a probability model for the number of male kittens you get.
b) What's the expected number of males?
c) What's the standard deviation?

23. Random variables. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:

	Mean	SD
X	10	2
Y	20	5

a) 3X b) Y + 6 c) X + Y d) X - Y

24. Random variables. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:

	Mean	SD
X	80	12
Y	12	3

a) X - 20
b) 0.5Y
c) X + Y
d) X - Y

25. Random variables. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:

	Mean	SD
X	120	12
Y	300	16

a) 0.8Y
b) 2X - 100
c) X + 2Y
d) 3X - Y

26. Random variables. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:

	Mean	SD
X	80	12
Y	12	3

a) 2Y + 20
b) 3X
c) 0.25X + Y
d) X - 5Y

27. Eggs. A grocery supplier believes that in a dozen eggs, the mean number of broken ones is 0.6 with a standard deviation of 0.5 eggs. You buy 3 dozen eggs without checking them.
a) How many broken eggs do you expect to get?
b) What's the standard deviation?

c) What assumptions did you have to make about the eggs in order to answer this question?

28. Garden. A company selling vegetable seeds in packets of 20 estimates that the mean number of seeds that will actually grow is 18, with a standard deviation of 1.2 seeds. You buy 5 different seed packets.
a) How many bad seeds do you expect to get?
b) What's the standard deviation?
c) What assumptions did you make about the seeds? Do you think that assumption is warranted? Explain.

29. Repair calls. Find the mean and standard deviation of the number of repair calls the appliance shop in Exercise 15 should expect during an 8-hour day.

30. Stop! Find the mean and standard deviation of the number of red lights the commuter in Exercise 16 should expect to hit on her way to work during a 5-day work week.

31. Fire! An insurance company estimates that it should make an annual profit of $150 on each homeowner's policy written, with a standard deviation of $6000.
a) Why is the standard deviation so large?
b) If it writes only two of these policies, what are the mean and standard deviation of the annual profit?
c) If it writes 10,000 of these policies, what are the mean and standard deviation of the annual profit?
d) Do you think the company is likely to be profitable? Explain,
e) What assumptions underlie your analysis? Can you think of circumstances under which those assumptions might be violated? Explain.

32. Casino. A casino knows that people play the slot machines in hopes of hitting the jackpot, but that most of them lose their dollar. Suppose a certain machine pays out an average of $0.92, with a standard deviation of $120.
a) Why is the standard deviation so large?
b) If you play 5 times, what are the mean and standard deviation of the casino's profit?
c) If gamblers play this machine 1000 times in a day, what are the mean and standard deviation of the casino's profit?
d) Do you think the casino is likely to be profitable? Explain.

33. Cereal. The amount of cereal that can be poured into a small bowl varies with a mean of 1.5 ounces and a standard deviation of 0.3 ounces. A large bowl holds a mean of 2.5 ounces with a standard deviation of 0.4 ounces. You open a new box of cereal and pour one large and one small bowl,
a) How much more cereal do you expect to be in the large bowl?
b) What's the standard deviation of this difference?
c) If the difference follows a Normal model, what's the probability the small bowl contains more cereal than the large one?
d) What are the mean and standard deviation of the total amount of cereal in the two bowls?
e) If the total follows a Normal model, what's the probability you poured out more than 4.5 ounces of cereal in the two bowls together?
f) The amount of cereal the manufacturer puts in the boxes is a random variable with a mean of 16.3 ounces and a standard deviation of 0.2 ounces. Find the expected amount of cereal left in the box, and the standard deviation.

34. Pets, The American Veterinary Association claims that the annual cost of medical care for dogs averages $100 with a standard deviation of $30, and for cats averages $120 with a standard deviation of $35.
a) What's the expected difference in the cost of medical care for dogs and cats?
b) What's the standard deviation of that difference?
c) If the difference in costs can be described by a Normal model, what's the probability that medical expenses are higher for someone's dog than for her cat?

35. More cereal. In Exercise 33 we poured a large and a small bowl of cereal from a box. Suppose the amount of cereal that the manufacturer puts in the boxes is a random variable with mean 16.2 ounces, and standard deviation 0.1 ounces.
a) Find the expected amount of cereal left in the box.
b) What's the standard deviation?
c) If the weight of the remaining cereal can be described by a Normal model, what's the probability that the box still contains more than 13 ounces?

36. More pets. You're thinking about getting two dogs and a cat. Assume that annual veterinary expenses are independent and have a Normal model with the means and standard deviations described in Exercise 34.
a) Define appropriate variables and express the total annual veterinary costs you may have,
b) Describe the model for this total cost. Be sure to specify its name, expected value, and standard deviation.
c) What's the probability that your total expenses will exceed $400?

37. Medley, in the 4 X 100 medley relay event, four swimmers swim 100 yards, each using a different stroke. A college team preparing for the conference championship looks at the times their swimmers have

posted and creates a model based on the following assumptions:

- The swimmers' performances are independent.
- Each swimmer's times follow a Normal model.
- The means and standard deviations of the times (in seconds) are as shown:

Swimmer	Mean	St. Dev.
1 (backstroke)	50.72	0.24
2 (breaststroke)	55.51	0.22
3 (butterfly)	49.43	0.25
4 (freestyle)	44.91	0.21

a) What are the mean and standard deviation for the relay team's total time in this event?
b) The team's best time so far this season was 3:19.48. (That's 199.48 seconds,) Do you think the team is likely to swim faster than this at the conference championship? Explain.

38. Bikes. Bicycles arrive at a bike shop in boxes. Before they can be sold, they must be unpacked, assembled, and tuned (lubricated, adjusted, etc.). Based on past experience, the shop manager makes the following assumptions about how long this may take:

- The times for each setup phase are independent.
- The times for each phase follow a Normal model.
- The means and standard deviations of the times (in minutes) are as shown:

Phase	Mean	St. Dev.
Unpacking	3.5	0.7
Assembly	21.8	2.4
Tuning	12.3	2.7

a) What are the mean and standard deviation for the total bicycle setup time?
b) A customer decides to buy a bike like one of the display models, but wants a different color. The shop has one, still in the box. The manager says they can have it ready in half an hour. Do you think the bike will be set up and ready to go as promised? Explain.

39. Farmers' market. A farmer has 100 lb of apples and 50 lb of potatoes for sale. The market price for apples (per pound) each day is a random variable with a mean of 0.5 dollars and a standard deviation of 0.2 dollars. Similarly, for a pound of potatoes, the mean price is 0.3 dollars and the standard deviation is 0.1 dollars. It also costs him 2 dollars to bring all the apples and potatoes to the market. The market is busy with eager shoppers, so we can assume that he'll be able to sell all of each type of produce at that day's price.
a) Define your random variables, and use them to express the farmer's net income.
b) Find the mean.
c) Find the standard deviation of the net income.
d) Do you need to make any assumptions in calculating the mean? How about the standard deviation?

40. Bike sale. The Exercise 38 bicycle shop will be offering 2 specially priced children's models at a sidewalk sale. The basic model will sell for $120 and the deluxe model for $150. Past experience indicates that sales of the basic model will have a mean of 5.4 bikes with a standard deviation of 1.2, and sales of the deluxe model will have a mean of 3.2 bikes with a standard deviation of 0.8 bikes. The cost of setting up for the sidewalk sale is $200.
a) Define random variables and use them to express the bicycle shop's net income.
b) What's the mean of the net income?
c) What's the standard deviation of the net income?
d) Do you need to make any assumptions in calculating the mean? How about the standard deviation?

Answers

1. a) 19 b) 4.2
2. a) 1.2 b) 280
3. a)

Win	$0	$5	$10	$30
P(amount won)	$\frac{26}{52}$	$\frac{13}{52}$	$\frac{12}{52}$	$\frac{1}{52}$

b) $4.13
c) $4 or less (answers may vary)

4. a)

Win	$100	$50	$0
P(amount won)	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{36}$

b) $23.61
c) $23 or less (answers may vary)

5. a)

Kids	1	2	3
P(kids)	0.5	0.25	0.25

b) 1.75
c) 0.875

Boys	0	1	2	3
P(boys)	0.5	0.25	0.125	0.125

6. a)

Won	$95	$90	$85	$80	-$20
P(amount won)	0.10	0.09	0.081	0.073	0.656

b) 3.44
c) $17.20

7. $27,000
8. $10,600
9. a) 7 b) 1.89
10. a) 0.75 b) 87.18
11. $5.44
12. $38.16
13. 0.83
14. $51.48
15. a) 1.7 b) 0.9
16. a) 2.25 b) 1.26
17. $\mu = 0.64$, $\sigma = 0.93$
18. a)

Profit	$100	-$9900	-$2900
P(profit)	0.9975	0.0005	0.002

b) $89.00
c) $260.54

19. a) No. The probability of winning the second depends on the outcome of the first.
b) 0.42
c) 0.08
d)

Games Won	0	1	2
P(games won)	0.42	0.50	0.08

e) $\mu = 0.66$, $\sigma = 0.62$

20. a) No. The chance to get the second contract depends on whether your company got the first.
b) 0.16
c) 0.14
d)

Contracts Won	0	1	2
P(contracts won)	0.14	0.70	0.16

e) $\mu = 1.02$, $\sigma = 0.55$

21. a)

Number good	0	1	2
P(numbers good)	0.067	0.467	0.466

b) 1.40
c) 0.61

22. a)

Number males	0	1	2
P(numbers males)	0.143	0.571	0.286

b) 1.14
c) 0.64

23. a) $\mu = 30$, $\sigma = 6$ b) $\mu = 26$, $\sigma = 5$
c) $\mu = 30$, $\sigma = 5.39$ d) $\mu = -10$, $\sigma = 5.39$
24. a) $\mu = 60$, $\sigma = 12$ b) $\mu = 6$, $\sigma = 1.50$
c) $\mu = 92$, $\sigma = 12.37$ d) $\mu = 68$, $\sigma = 12.37$
25. a) $\mu = 240$, $\sigma = 12.80$ b) $\mu = 140$, $\sigma = 24$
c) $\mu = 720$, $\sigma = 34.18$ d) $\mu = 60$, $\sigma = 39.4$
26. a) $\mu = 44$, $\sigma = 6$ b) $\mu = 240$, $\sigma = 36$
c) $\mu = 32$, $\sigma = 4.24$ d) $\mu = 20$, $\sigma = 19.21$
27. a) 1.8
b) 0.87
c) Cartons are independent of each other.
28. a) 10
b) 2.68
c) Packets are independent of each other. OK if different types of seeds; if all the same type (and lot), assumption would probably not be valid.
29. $\mu = 13.6$, $\sigma = 2.55$ (assuming the hours are independent of each other).
30. $\mu = 11.25$, $\sigma = 2.82$
31. a) There will be many gains of $150 with a few large losses.
b) $\mu = \$300$, $\sigma = \$8485.28$
c) $\mu = \$1,500,000$, $\sigma = \$600,000$
d) Yes. $0 is 2.5 SD below the mean for 10,000 policies.
e) Losses are independent of each other. A major catastrophe with many policies in an area would violate the assumption.

32. a) Gamblers lose a relatively small amount most of the time, but there are a few large payouts.
   b) μ = \$0.40, σ = \$268.33
   c) μ = \$80.00, σ = 3794.73
   d) If the machine is played only 1000 times a day, the chance of being profitable will be slightly more than 50%, since \$80 is about 0.02 SDs above 0. But if the casino has many slot machines, the chances of being profitable will go up.
33. a) 1 oz b) 0.5 oz c) 0.023
   d) μ = 4 oz, σ = 0.5 oz
   e) 0.159
   f) μ = 12.3 oz, σ = 0.54 oz

34. a) -\$20 b) \$46.10 c) 0.332
35. a) 12.2 oz b) 0.51 oz c) 0.058
36. a) X = cost for a dog; Y = cost for a cat; total costs = $X_1 + X_2 + Y$
   b) Normal, μ = \$320, σ = \$55
   c) 0.073
37. a) μ = 200.57 sec, σ = 0.46 sec
   b) No, $Z = \frac{199.48 - 200.57}{0.461}$ = -2.36. There is only 0.009 probability of swimming that fast or faster.
38. a) μ = 37.6 min, σ = 3.7 min
   b) No, 30 min is more than 2 SDs below the mean.
39. a) A = price of a pound of apples; p = price of a pound of potatoes; Profit = 100A + 50P - 2
   b) \$63.00
   c) \$20.62
   d) Mean — no; SD — yes (independent sales prices).
40. a) B = number basic; D = number deluxe; Net = 120B + 150D - 200
   b) \$928.00
   c) \$187.45
   d) Mean — no; SD — yes (sales are independent).