1. Sample spaces. For each of the following, list the sample space and tell whether you think the outcomes are equally likely.
a) Toss 2 coins; record the order of heads and tails.
b) A family has 3 children; record the number of boys.
c) Flip a coin until you get a head or 3 consecutive tails.
d) Roll two dice; record the larger number.
2. Sample spaces. For each of the following, list the sample space and tell whether you think the outcomes are equally likely.
a) Roll two dice; record the sum of the numbers.
b) A family has 3 children; record the genders in order of birth.
c) Toss four coins; record the number of tails.
d) Toss a coin 10 times; record the longest run of heads.
3. Homes. Real estate ads suggest that $64 \%$ of homes for sale have garages, $21 \%$ have swimming pools, and $17 \%$ have both features. What is the probability that a home for sale has
a) a pool or a garage?
b) neither a pool nor a garage?
c) a pool but no garage?
4. Travel. Suppose the probability that a U.S. resident has traveled to Canada is 0.18 , to Mexico is 0.09 , and to both countries is 0.04 . What's the probability that an American chosen at random has
a) traveled to Canada but not Mexico?
b) traveled to either Canada or Mexico?
c) not traveled to either country?
5. Amenities. A check of dorm rooms on a large college campus revealed that $38 \%$ had refrigerators, $52 \%$ had TVs, and $21 \%$ had both a TV and a refrigerator. What's the probability that a randomly selected dorm room has
a) a TV but no refrigerator?
b) a TV or a refrigerator, but not both?
c) neither a TV nor a refrigerator?
6. Workers. Employment data at a large company reveal that $72 \%$ of the workers are married, that $44 \%$ are college graduates, and that half of the college grads are married. What's the probability that a randomly chosen worker
a) is neither married nor a college graduate?
b) is married but not a college graduate?
c) is married or a college graduate?
7. First lady. A Gallup survey of June 2004 asked 1005 U.S. adults who they think better fits their idea of what a first lady should be, Laura Bush or Hillary Rodham Clinton. Suppose the data break down as follows:

|  |  | Age Group |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 18-29 | 30-49 | 50-64 | Over 65 | Total |
| $\begin{aligned} & \mathscr{H} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Clinton | 135 | 158 | 79 | 65 | 437 |
|  | Bush | 77 | 237 | 112 | 92 | 518 |
|  | Equally/Neither/ <br> No opinion | 3 | 21 | 14 | 10 | 50 |
|  | Total | 217 | 416 | 205 | 167 | 1005 |

If we select a person at random from this sample:
a) What is the probability that the person thought Laura Bush best fits their first lady ideal?
b) What is the probability that the person is younger than 50 years old?
c) What is the probability that the person is younger than 50 and thinks Hillary Clinton best fits their ideal?
d) What is the probability that the person is younger than 50 or thinks Hillary Clinton best fits their ideal?
8. Birth order. A survey of students in a large Introductory Statistics class asked about their birth order (first or only child, second, etc.) and which college of the university they were enrolled in. Here are the data:

|  |  | Birth Order |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | First or Only | Second or later | Total |
| $\begin{aligned} & \stackrel{0}{80} \\ & \stackrel{0}{0} \end{aligned}$ | Arts \& Sciences | 34 | 23 | 57 |
|  | Agriculture | 52 | 41 | 93 |
|  | Human Ecology | 15 | 28 | 43 |
|  | Other | 12 | 18 | 30 |
|  | Total | 113 | 110 | 223 |

Suppose we select a student at random from this class.
a) What is the probability we select a Human Ecology student?
b) What is the probability that we select a first-born student?
c) What is the probability that the person is firstborn and a Human Ecology student?
d) What is the probability that the person is firstborn or a Human Ecology student?
9. Cards. You draw a card at random from a standard deck of 52 cards. Find each of the following conditional probabilities:
a) The card is a heart, given that it is red.
b) The card is red, given that it is a heart.
c) The card is an ace, given that it is red.
d) The card is a queen, given that it is a face card.
10. Pets. In its monthly report, the local animal shelter states that it currently has 24 dogs and 18 cats available for adoption. Eight of the dogs and 6 of the cats are male. Find each of the following conditional probabilities if an animal is selected at random:
a) The pet is male, given that it is a cat.
b) The pet is a cat, given that it is female.
c) The pet is female, given that it is a dog.
11. Health. The probabilities that an adult American man has high blood pressure and/ or high cholesterol are shown in the table.

|  |  | Blood Pressure |  |
| :---: | :---: | :---: | :---: |
|  |  | High | OK |
|  | High | 0.11 | 0.21 |
|  | OK | 0.16 | 0.52 |

a) What's the probability that a man has both conditions?
b) What's the probability that he has high blood pressure?
c) What's the probability that a man with high blood pressure has high cholesterol?
d) What's the probability that a man has high blood pressure if it's known that he has high cholesterol?
12. Death penalty. The table shows the political affiliation of American voters and their positions on the death penalty.

|  |  | Death Penalty |  |
| :---: | :---: | :---: | :---: |
|  | Favor | Oppose |  |
|  | Republican | 0.26 | 0.04 |
| 空 | Democrat | 0.12 | 0.24 |
| Other | 0.24 | 0.10 |  |

a) What's the probability that a randomly chosen voter favors the death penalty?
b) What's the probability that a Republican favors the death penalty?
c) What's the probability that a voter who favors the death penalty is a Democrat?
d) A candidate thinks she has a good chance of gaining the votes of anyone who is a Republican or in favor of the death penalty. What portion of the voters is that?
13. First lady, take 2. Look again at the data from the Gallup survey on first ladies in Exercise 7.
a) If we select a respondent at random, what's the probability we choose a person between 18 and 29 who picked Clinton?
b) Among the 18 - to 29 -year-olds, what is the probability that a person responded "Clinton"?
c) What's the probability that a person who chose Clinton was between 18 and 29?
d) If the person responded "Bush," what is the probability that they are over 65 ?
e) What's the probability that a person over 65 preferred Bush?
14. Birth order, take 2. Look again at the data about birth order of Intro Stats students and their choices of colleges shown in Exercise 8.
a) If we select a student at random, what's the probability the person is an Arts and Sciences student who is a second child (or more)?
b) Among the Arts and Sciences students, what's the probability a student was a second child (or more)?
c) Among second children (or more), what's the probability the student is enrolled in Arts and Sciences?
d) What's the probability that a first or only child is enrolled in the Agriculture College?
e) What is the probability that an Agriculture student is a first or only child?
15. Sick kids. Seventy percent of kids who visit a doctor have a fever, and $30 \%$ of kids with a fever have sore throats. What's the probability that a kid who goes to the doctor has a fever and a sore throat?
16. Sick cars. Twenty percent of cars that are inspected have faulty pollution control systems. The cost of repairing a pollution control system exceeds $\$ 100$ about $40 \%$ of the time. When a driver takes her car in for inspection, what's the probability that she will end up paying more than $\$ 100$ to repair the pollution control system?
17. Cards. You are dealt a hand of three cards, one at a time. Find the probability of each of the following.
a) The first heart you get is the third card dealt.
b) Your cards are all red (that is, all diamonds or hearts).
c) You get no spades.
d) You have at least one ace.
18. Another hand. You pick three cards at random from a deck. Find the probability of each event described below.
a) You get no aces.
b) You get all hearts.
c) The third card is your first red card.
d) You have at least one diamond.
19. Batteries. A junk box in your room contains a dozen old batteries, five of which are totally dead. You start picking batteries one at a time and testing them. Find the probability of each outcome.
a) The first two you choose are both good.
b) At least one of the first three works.
c) The first four you pick all work.
d) You have to pick 5 batteries in order to find one that works.
20. Shirts. The soccer team's shirts have arrived in a big box, and people just start grabbing them, looking for the right size. The box contains 4 medium, 10 large, and 6 extra large shirts. You want a medium for you and one for your sister. Find the probability of each event described.
a) The first two you grab are the wrong sizes.
b) The first medium shirt you find is the third one you check.
c) The first four shirts you pick are all extra-large.
d) At least one of the first four shirts you check is a medium.
21. Eligibility. A university requires its biology majors to take a course called BioResearch. The prerequisite for this course is that students must have taken either a Statistics course or a computer course. By the time they are juniors, $52 \%$ of the Biology majors have taken Statistics, $23 \%$ have had a computer course, and $7 \%$ have done both.
a) What percent of the junior Biology majors are ineligible for BioResearch?
b) What's the probability that a junior Biology major who has taken Statistics has also taken a computer course?
c) Are taking these two courses disjoint events? Explain.
d) Are taking these two courses independent events? Explain.
22. Benefits. Fifty-six percent of all American workers have a workplace retirement plan, $68 \%$ have health insurance, and $49 \%$ have both benefits. We select a worker at random.
a) What's the probability he has neither employer sponsored health insurance nor a retirement plan?
b) What's the probability he has health insurance if he has a retirement plan?
c) Are having health insurance and a retirement plan independent events? Explain.
d) Are having these two benefits mutually exclusive? Explain.
23. For sale. In the real estate ads described in Exercise $3,64 \%$ of homes for sale have garages, $21 \%$ have swimming pools, and $17 \%$ have both features.
a) If a home for sale has a garage, what's the probability that it has a pool, too?
b) Are having a garage and a pool independent events? Explain.
c) Are having a garage and a pool mutually exclusive? Explain.
24. On the road again. According to Exercise 4, the probability that a U.S. resident has traveled to Canada is 0.18 , to Mexico is 0.09 , and to both countries is 0.04 .
a) What's the probability that someone who has traveled to Mexico has visited Canada, too?
b) Are travel to Mexico and Canada disjoint events? Explain.
c) Are travel to Mexico and Canada independent events? Explain.
25. Cards. If you draw a card at random from a well shuffled deck, is getting an ace independent of the suit? Explain.
26. Pets again. The local animal shelter in Exercise 10 reported that it currently has 24 dogs and 18 cats available for adoption; 8 of the dogs and 6 of the cats are male. Are the species and gender of the animals independent? Explain.
27. First lady, final visit. In Exercises 7 and 13 we looked at results of a Gallup Poll that asked people whether they thought Laura Bush or Hillary Clinton better fits their idea of a first lady.
a) Are being under 30 and being over 65 disjoint? Explain.
b) Are being under 30 and being over 65 independent? Explain.
c) Are answering "Clinton" and being over 65 disjoint? Explain.
d) Are answering "Clinton" and being over 65 independent? Explain.
28. Birth order, finis. In Exercises 8 and 14 we looked at the birth orders and college choices of some Intro Stats students.
a) Are enrolling in Agriculture and Human Ecology disjoint? Explain.
b) Are enrolling in Agriculture and Human Ecology independent? Explain.
c) Are being first-born and enrolling in Human Ecology disjoint? Explain.
d) Are being first-born and enrolling in Human Ecology independent? Explain.
29. Men's health, again. Given the table of probabilities from Exercise 11, are high blood pressure and high cholesterol independent? Explain.

| $\begin{aligned} & \overline{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | Blood Pressure |  |
| :---: | :---: | :---: | :---: |
|  |  | High | OK |
|  | High | 0.11 | 0.21 |
|  | OK | 0.16 | 0.52 |

30. Politics. Given the table of probabilities from

Exercise 12, are party affiliation and position on the death penalty independent? Explain.

|  | Death Penalty |  |
| :---: | :---: | :---: |
|  | Favor | Oppose |
|  | Republican | 0.26 |
| 元 | 0.04 |  |
| Democrat | 0.12 | 0.24 |
| Other | 0.24 | 0.10 |

31. Phone service. According to estimates from the federal government's 2003 National Health Interview Survey, based on face-to-face interviews in 16,677 households, approximately $58.2 \%$ of U.S. adults have both a land line in their residence and a cell phone, $2.8 \%$ have only cell phone service but no land line, and $1.6 \%$ have no telephone service at all.
a) Polling agencies won't phone cell phone numbers because customers object to paying for such calls. What proportion of U.S. households can be reached by a land line call?
b) Are having a cell phone and having a land line independent? Explain.
32. Snoring. After surveying 995 adults, $81.5 \%$ of whom were over 30, the National Sleep Foundation reported that $36.8 \%$ of all the adults snored. $32 \%$ of the respondents were snorers over the age of 30 .
a) What percent of the respondents were under 30 and did not snore?
b) Is snoring independent of age? Explain.
33. Montana. A 1992 poll conducted by the University of Montana classified respondents by gender and political party, as shown in the table. Is party affiliation independent of sex? Explain.

|  | Democrat | Republican | Independent |
| :--- | :---: | :---: | :---: |
| Male | 36 | 45 | 24 |
| Female | 48 | 33 | 16 |

34. Cars. A random survey of autos parked in student and staff lots at a large university classified the brands by country of origin, as seen in the table. Is country of origin independent of type of driver?

| 品 |  | Driver |  |
| :---: | :---: | :---: | :---: |
|  |  | Student | Staff |
|  | American | 107 | 105 |
| $\bigcirc$ | European | 33 | 12 |
|  | Asian | 55 | 47 |

35. Luggage. Leah is flying from Boston to Denver with a connection in Chicago. The probability her first flight leaves on time is 0.15 . If the flight is on time, the probability that her luggage will make the connecting flight in Chicago is 0.95 , but if the first
flight is delayed, the probability that the luggage will make it is only 0.65 .
a) Are the first flight leaving on time and the luggage making the connection independent events? Explain.
b) What is the probability that her luggage arrives in Denver with her?
36. Graduation. A private college report contains these statistics:

70\% of incoming freshmen attended public schools.
$75 \%$ of public school students who enroll as freshmen eventually graduate. $90 \%$ of other freshmen eventually graduate.
a) Is there any evidence that a freshman's chances to graduate may depend upon what kind of high school the student attended? Explain.
b) What percent of freshmen eventually graduate?
37. Late luggage. Remember Leah (Exercise 35)? Suppose you pick her up at the Denver airport, and her luggage is not there. What is the probability that Leah's first flight was delayed?
38. Graduation, part II. What percent of students who graduate from the college in Exercise 36 attended a public high school?
39. Absenteeism. A company's records indicate that on any given day about $1 \%$ of their day shift employees and $2 \%$ of the night shift employees will miss work. Sixty percent of the employees work the day shift.
a) Is absenteeism independent of shift worked? Explain.
b) What percent of employees are absent on any given day?
40. Lungs and smoke. Suppose that $23 \%$ of adults smoke cigarettes. It's known that $57 \%$ of smokers and $13 \%$ of nonsmokers develop a certain lung condition by age 60 .
a) Explain how these statistics indicate that lung condition and smoking are not independent.
b) What's the probability that a randomly selected 60 -year-old has this lung condition?
41. Absenteeism, part II. At the company described in Exercise 39, what percent of the absent employees are on the night shift?
42. Lungs and smoke, again. Based on the statistics in Exercise 40, what's the probability that someone with the lung condition was a smoker?
43. Drunks. Police often set up sobriety checkpoints roadblocks where drivers are asked a few brief questions to allow the officer to judge whether or not the person may have been drinking. If the officer does not suspect a problem, drivers are released to go on their way. Otherwise, drivers are detained for a Breathalyzer test that will determine whether or not they are arrested. The police say that based on the brief initial stop, trained officers can make the right decision $80 \%$ of the time. Suppose the police operate a sobriety checkpoint after $9 \mathrm{p} . \mathrm{m}$. on a Saturday night, a time when national traffic safety experts suspect that about $12 \%$ of drivers have been drinking.
a) You are stopped at the checkpoint and, of course, have not been drinking. What's the probability that you are detained for further testing?
b) What's the probability that any given driver will be detained?
c) What's the probability that a driver who is detained has actually been drinking?
d) What's the probability that a driver who was released had actually been drinking?
44. Polygraphs. Lie detectors are controversial instruments, barred from use as evidence in many courts. Nonetheless, many employers use lie detector screening as part of their hiring process in the hope that they can avoid hiring people who might be dishonest. There has been some research, but no agreement, about the reliability of polygraph tests. Based on this research, suppose that a polygraph can detect $65 \%$ of lies, but incorrectly identifies $15 \%$ of true statements as lies.
A certain company believes that $95 \%$ of its job applicants are trustworthy. The company gives everyone a polygraph test, asking, "Have you ever stolen anything from your place of work?" Naturally, all the applicants answer "No," but the polygraph identifies some of those answers as lies, making the person ineligible for a job. What's the probability that a job applicant rejected under suspicion of dishonesty was actually trustworthy?
45. Dishwashers. Dan's Diner employs three dishwashers. Al washes $40 \%$ of the dishes and breaks only $1 \%$ of those he handles. Betty and Chuck each wash $30 \%$ of the dishes, and Betty breaks only $1 \%$ of hers, but Chuck breaks $3 \%$ of the dishes he washes. (He, of course, will need a new job soon ... . ) You go to Dan's for supper one night and hear a dish break at the sink. What's the probability that Chuck is on the job?

Answers

1. a) $\mathrm{S}=\{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$ All of the outcomes are equally likely to occur.
b) $\mathrm{S}=\{0,1,2,3\}$ All outcomes are not equally likely. A family of 3 is more likely to have, for example, 2 boys than 3 boys. There are three equally likely outcomes that result in 2 boys ( $\mathrm{BBG}, \mathrm{BGB}$, and GBB), and only one that results in 3 boys ( BBB ).
c) $S=\{\mathrm{H}, \mathrm{TH}, \mathrm{TTH}, \mathrm{TTT}\}$ All outcomes are not equally likely. For example the probability of getting heads on the first try is $\frac{1}{2}$. The probability of getting three tails is $\left(\frac{1}{2}\right)^{3}=\frac{1}{8}$
d) $S=\{1,2,3,4,5,6\}$ All outcomes are not equally likely. Since you are recording only the larger number of two dice, 6 will be the larger when the other die reads $1,2,3,4$, or 5 . The outcome 2 will only occur when the other die shows 1 or 2 .
2. a) $\mathrm{S}=\{2,3,4,5,6,7,8,9,10,11,12\}$ All outcomes are not equally likely. For example, there are four equally likely outcomes that result in a sum of $5(1+4,4+1,2+$ 3 , and $3+2$ ), and only one outcome that results in a sum of $2(1+1)$.
b) $\mathrm{S}=\{\mathrm{BBB}, \mathrm{BBG}, \mathrm{BGB}, \mathrm{BGG}, \mathrm{GBB}, \mathrm{GBG}, \mathrm{GGB}, \mathrm{GGG}\}$ All outcomes are equally likely.
c) $\mathrm{S}=\{0,1,2,3,4\}$ All outcomes are not equally likely. For example, there are 4 equally likely outcomes that produce 1 tail (HHHT, HHTH, HTHH, and THHH), but only one outcome that produces 4 tails (TTTT).
d) $S=\{0,1,2,3,4,5,6,7,8,9,10\}$ All outcomes are not equally likely. A string of 3 heads is much more likely to occur than a string of 10 heads in a row.
3. Construct a Venn diagram of the disjoint outcomes.

a) $P($ pool $\cup$ garage $)=P($ pool $)+P($ garage $)-P($ pool $\cap$ garage $)=0.64+0.21-0.17=0.68$
Or, from the Venn: $0.47+0.17+0.04=0.68$
b) $P($ neither $)=1-P($ pool $\cup$ garage $)=1-0.68=0.32$

Or, from the Venn: 0.32 (the region outside the circles)
c) $P($ pool $\cap$ no garage $)=P($ pool $)-P($ pool $\cap$ garage $)=$ $0.21-0.17=0.04$
Or, from the Venn: 0.04 (the region inside pool circle, yet outside garage circle)
4. Construct a Venn diagram of the disjoint outcomes.

a) $P($ Canada $\cap$ not Mexico $)=P($ Canada $)-P($ Canada $\cap$

Mexico) $=0.18-0.04=0.14$
Or, from the Venn: 0.14 (region inside the Canada circle, yet outside the Mexico circle)
b) $P($ either Canada $\cup$ Mexico $)=P($ Canada $)+P($ Mexico $)-$
$P($ Canada $\cap$ Mexico $)=0.18+0.09-0.04=0.23$
Or, from the Venn: $0.05+0.04+0.14=0.23$ (the regions inside the circles)
c) $P($ neither Canada nor Mexico $)=1-P($ either Canada $\cup$

Mexico) $=1-0.23=0.77$
Or, from the Venn: 0.77 (the region outside the circles)
5. Construct a Venn diagram of the disjoint outcomes.

a) $P(\mathrm{TV} \cap$ no refrigerator $)=P(\mathrm{TV})-P(\mathrm{TV} \cap$ refrigerator $)$ $=0.52-0.21=0.31$
Or, from the Venn: 0.31 (the region inside the TV circle, yet outside the Fridge circle)
b) $P($ refrigerator $\cup$ TV, but not both $)=[P($ refrigerator $)-$ $P($ refrigerator $\cap \mathrm{TV})]+[P(\mathrm{TV})-P($ refrigerator $\cap \mathrm{TV})]=$ $[0.38-0.21]+[0.52-0.21]=0.48$
This problem is much easier to visualize using the Venn diagram. Simply add the probabilities in the two regions for Fridge only and TV only.
$P($ refrigerator $\cup$ TV, but not both $)=0.17+0.31=0.48$
c) $P($ neither TV nor refrigerator $)=1-P($ either TV $\cup$ refrigerator $)=1-[P(\mathrm{TV})+P($ refrigerator $)-P(\mathrm{TV} \cap$ refrigerator $)]=1-[0.52+0.38-0.21]=0.31$
Or, from the Venn: 0.31 (the region outside the circles)
6. Construct a Venn diagram of the disjoint outcomes.

a) $P($ neither married nor a college graduate $)=1-P($ either married $\cup$ college graduate $)=1-[P($ married $)+P($ college graduate $)-P($ both $)]=1-[0.72+0.44-0.22]=1-[0.94]$ $=0.06$
Or, from the Venn: 0.06 (the region outside the circles) b) $P($ married $\cap$ not a college graduate $)=P($ married $)-$ $P($ married $\cap$ a college graduate $)=0.72-0.22=0.50$ Or, from the Venn: 0.50 (the region inside the Married circle, yet inside the College circle)
c) $P($ married $\cup$ a college graduate $)=P($ married $)+$ $P($ college graduate $)-P($ both $)=0.72+0.44-0.22=0.94$ Or, from the Venn diagram: $0.22+0.22+0.50=0.94$ (the regions inside the circles)
7. a) $P($ Laura Bush $)=\approx \frac{518}{1005}=0.515$
b) $P($ younger than 50 years $)=\frac{217+416}{1005}=0.630$
c) $P($ younger than $50 \cap$ Hillary Clinton $)=$
$\frac{135+158}{1005}=0.292$
d) $P($ younger than $50 \cup$ Hillary Clinton $)=P($ younger than $50)+P($ Clinton $)-P($ younger than $50 \cap$ Clinton $)=$ $\frac{217+416}{1005}+\frac{437}{1005}-\frac{135+158}{1005}=0.773$
8. a) $\frac{43}{223}$
b) $\frac{113}{223}$
c) $\frac{15}{223}$
d) $\frac{113}{223}+\frac{43}{223}-\frac{15}{223}=\frac{141}{223}$
9. a) $\frac{P(\text { heart } \cap \text { red })}{P(\text { red })}=\frac{13 / 52}{26 / 52}=\frac{1}{2}$
b) $\frac{P(\text { red } \cap \text { heart })}{P(\text { heart })}=\frac{13 / 52}{13 / 52}=1$
c) $\frac{P(\text { ace } \cap \text { red })}{P(\text { red })}=\frac{2 / 52}{26 / 52}=\frac{1}{13}$
d) $\frac{P(\text { queen } \cap \text { face })}{P(\text { face })}=\frac{4 / 52}{12 / 52}=\frac{1}{3}$
10. Organize the counts in a two-way table.

|  | Cats | Dogs | Total |
| :--- | :--- | :--- | :--- |
| Male | 6 | 8 | 14 |
| Female | 12 | 16 | 28 |
| Total | 18 | 24 | 42 |

a) $\frac{6 / 42}{18 / 42}=\frac{1}{3}$ Consider only the Cats column. There are 6 male cats, out of a total of 18 cats.
b) $\frac{12 / 42}{28 / 42}=\frac{3}{7}$ We are interested in the Female row. Of the 28 female animals, 12 are cats.
c) $\frac{16 / 42}{24 / 42}=\frac{2}{3}$ Look at only the Dogs column. There are 24 dogs, and 16 of them are female.
11. Construct a two-way table of the conditional probabilities, including the marginal probabilities.

Blood Pressure

|  |  | High | OK | Total |
| :---: | :---: | :---: | :---: | :---: |
|  | High | 0.11 | 0.21 | 0.32 |
|  | OK | 0.16 | 0.52 | 0.68 |
|  | Total | 0.27 | 0.73 | 1.00 |

a) $P($ both conditions $)=0.11$
b) $P($ high blood pressure $)=0.11+0.16=0.27$
c) $\frac{0.11}{0.27}=0.407$ Consider only the High Blood Pressure column. Within this column, the probability of having high cholesterol is 0.11 out of a total of 0.27 .
d) $\frac{0.11}{0.32}=0.344$ This time, consider only the high
cholesterol row. Within this row, the probability of having high blood pressure is 0.11 , out of a total of 0.32 .
12. Construct a two-way table of the conditional probabilities, including the marginal probabilities.
a) $P($ favor the death penalty $)=0.26+0.12+0.24=0.62$
b) $\frac{0.26}{0.30}=0.867$ Consider only the Republican row. The probability of favoring the death penalty is 0.26 out of a total of 0.30 for that row.
c) $\frac{0.12}{0.62}=0.194$ Consider only the Favor column. The probability of being a Democrat is 0.12 out of a total of 0.62 for that column.
d) $P($ Republican $\cup$ favor death penalty $)=P($ Republican $)+$ $P($ favor death pen. $)-P($ both $)=0.30+0.62-0.26=0.66$ The overall probabilities of being a Republican and favoring the death penalty are from the marginal
distribution of probability (the totals). The candidate can expect $66 \%$ of the votes, provided her estimates are correct.
13. First lady, take 2.
a) $P($ between 18 and 29 Clinton $)=\frac{135}{1005}$
b) $P($ Clinton $\mid$ between 18 and 29 $)=\frac{135}{217}$
c) $P($ between 18 and $29 \mid$ Clinton $)=\frac{135}{437}$
d) $P($ over $65 \mid$ Bush $)=\frac{92}{518}$
e) $P($ Bush $\mid$ over 65$)=\frac{92}{165}$
14. a) $P($ Arts and Science second child $)=\frac{23}{223}$
b) $P($ second child $\mid$ Arts and Science $)=\frac{23}{57}$
c) $P($ Arts and Science $\mid$ second child $)=\frac{23}{110}$
d) $P($ Agriculture $\mid$ first - born $)=\frac{52}{113}$
e) $P($ first - born $\mid$ Agriculture $)=\frac{52}{93}$
15. Having a fever and having a sore throat are not independent events, so: $P($ fever and sore throat $)=$ $P($ Fever $) P($ Sore Throat $\mid$ Fever $)=(0.70)(0.30)=0.21$
The probability that a kid with a fever has a sore throat is 0.21 .
16. Needing repairs and paying more than $\$ 400$ for the repairs are not independent events.
(What happens to the probability of paying more than $\$ 400$, if you don't need repairs?!)
$P($ needing repairs $\cap$ paying more than $\$ 400)=P($ needing repairs) $P$ (paying more than $\$ 400 \mid$ repairs are needed $)=$ $(0.20)(0.40)=0.08$
17. a) $\left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{13}{50}\right)=0.145$
b) $\left(\frac{26}{52}\right)\left(\frac{25}{51}\right)\left(\frac{24}{50}\right)=0.118$
c) $\left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right)=0.414$
d) $1-\left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right)=0.217$
18. a) $\left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right)=0.783$
b) $\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)=0.013$
c) $\left(\frac{26}{52}\right)\left(\frac{25}{51}\right)\left(\frac{26}{50}\right)=0.414$
d) $1-\left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right)=0.586$
19. Since batteries are not being replaced, use conditional probabilities throughout.
a) $\left(\frac{7}{12}\right)\left(\frac{6}{11}\right)=0.318$
b) $1-\left(\frac{5}{12}\right)\left(\frac{4}{11}\right)\left(\frac{3}{10}\right)=0.955$
c) $\left(\frac{7}{12}\right)\left(\frac{6}{11}\right)\left(\frac{5}{10}\right)\left(\frac{4}{9}\right)=0.071$
d) $\left(\frac{5}{12}\right)\left(\frac{4}{11}\right)\left(\frac{3}{10}\right)\left(\frac{2}{9}\right)\left(\frac{7}{8}\right)=0.009$
20. You need two shirts so don't replace them. Use conditional probabilities throughout.
a) $\left(\frac{16}{20}\right)\left(\frac{15}{19}\right)=0.632$
b) $\left(\frac{16}{20}\right)\left(\frac{15}{19}\right)\left(\frac{4}{18}\right)=0.140$
c) $\left(\frac{6}{20}\right)\left(\frac{5}{19}\right)\left(\frac{4}{18}\right)\left(\frac{3}{17}\right)=0.003$
d) $1-\left(\frac{16}{20}\right)\left(\frac{15}{19}\right)\left(\frac{14}{18}\right)\left(\frac{13}{17}\right)=0.624$
21. Construct a Venn diagram of the disjoint outcomes.

a) $P($ eligibility $)=P($ statistics $)$
$+P($ computer science $)-P($ both $)=0.2$
$+0.23-0.07=0.68$
$68 \%$ of students are eligible for
BioResearch, so $100-68=32 \%$ are ineligible.
From the Venn, the region outside the circles represents those students who have taken neither course, and are therefore ineligible for BioResearch.
b) $\frac{0.07}{0.52}=0.135$ From the Venn, consider only the region inside the Statistics circle. The probability of having taken computer science is 0.07 out of a total of 0.52 (the entire Statistics circle).
c) Taking the two courses are not disjoint events, since they have outcomes in common. In fact, $7 \%$ of juniors have taken both courses.
d) Taking the two courses are not independent events. The overall probability that a junior has taken a computer science is 0.23 . The probability that a junior has taken a computer course given that he or she has taken a statistics course is 0.135 . If taking the two courses were independent events, these probabilities would be the same.
22. Construct a Venn diagram of the disjoint outcomes.

a) $P($ neither benefit $)=1-P($ either retirement $U$ health $)=1-$ $[P($ retirement $)+P($ health $)-P($ both $)]$
$=1-[0.56+0.68-0.49]=0.25$
b) $\frac{0.49}{0.56}=0.875$ From the Venn, consider only the region inside the Retirement circle. The probability that a worker has health insurance is 0.49 out of a total of 0.56 (the entire Retirement circle).
c) Having health insurance and a retirement plan are not independent events. $68 \%$ of all workers have health
insurance, while $87.5 \%$ of workers with retirement plans also have health insurance. If having health insurance and a retirement plan were independent events, these percentages would be the same.
d) Having these two benefits are not disjoint events, since they have outcomes in common. $49 \%$ of workers have both health insurance and a retirement plan.
23. Construct a Venn diagram of the disjoint outcomes.

a) $\frac{0.17}{0.64}=0.266$ From the Venn, consider only the region inside the Garage circle. The
probability that the house has a pool is 0.17 out of a total of 0.64 (the entire Garage circle).
b) Having a garage and a pool are not independent events. $26.6 \%$ of homes with garages have pools. Overall, $21 \%$ of homes have pools. If having a garage and a pool were independent events, these would be the same.
c) No, having a garage and a pool are not disjoint events. $17 \%$ of homes have both.
24. Construct a Venn diagram of the disjoint outcomes.

a) $\frac{0.04}{0.09}=0.444$ From the Venn,
consider only the region inside the Mexico circle.
The probability that an American has traveled to Canada is 0.04 out of a total of 0.09 (the entire Mexico circle). b) No, travel to Mexico and Canada are not disjoint events. $4 \%$ of Americans have been to both countries.
c) No, travel to Mexico and Canada are not independent events. $18 \%$ of U.S. residents have been to Canada. $44.4 \%$ of the U.S. residents who have been to Mexico have also been to Canada. If travel to the two countries were independent, the percentages would be the same.
25. Yes, getting an ace is independent of the suit when drawing one card from a well shuffled deck. The overall probability of getting an ace is $4 / 52$, or $1 / 13$, since there are 4 aces in the deck. If you consider just one suit, there is only 1 ace out of 13 cards, so the probability of getting an ace given that the card is a diamond, for instance, is $1 / 13$. Since the probabilities are the same, getting an ace is independent of the suit.
26. Yes, species and gender are independent events. 8 of 24 , or $1 / 3$ of the dogs are male, and 6 of 18 , or $1 / 3$ of the cats are male. Since these are the same, species and gender are independent events.
27. a) Yes, since they share no outcomes. No one is both under 30 and over 65.
b) No, since knowing that one event is true drastically changes the probability of the other. The probability of a respondent chosen at random being under 30 is almost $22 \%$. The probability of being under 30 , given that the respondent is over 65 is 0 .
c) No, since the events share outcomes. There were 65 respondents who were over 65 and chose Clinton.
d) No, since knowing that one event is true drastically changes the probability of the other. Over $43 \%$ of all respondents chose Clinton, but only $39 \%$ of those over 65 did.
28. a) Yes, since the events share no outcomes. Students can enroll in only one college.
b) No, since knowing that one event is true drastically changes the probability of the other. The probability of a student being in the Agriculture college is nearly $42 \%$. The probability of a student being in the Human Ecology college, given that he or she is in the Agriculture college is 0 .
c) No, since they share outcomes. 15 students were firstborn, Human Ecology students.
d) No, since knowing that one event is true drastically changes the probability of the other. Over $19 \%$ of all students enrolled in Human Ecology, but only 13\% of first-borns did.
29. High blood pressure and high cholesterol are not independent events. $28.8 \%$ of men with OK blood pressure have high cholesterol, while $40.7 \%$ of men with high blood pressure have high cholesterol. If having high blood pressure and high cholesterol were independent, these percentages would be the same.
30. Party affiliation and position on the death penalty are not independent events. $86.7 \%$ of Republicans favor the death penalty, but only $33.3 \%$ of Democrats favor it. If the events were independent, then these percentages would be the same.
31. a) Since $2.8 \%$ of U.S. adults have only a cell phone, and $1.6 \%$ have no phone at all, polling organizations can reach $100-2.8-1.6=96.5 \%$ of U.S. adults. b) Using the Venn diagram, about $96.5 \%$ of U.S. adults have a land line. The probability of a U.S. adults having a land line given that they have a cell phone is $58.2 /(58.2+2.8)$ or about $95.4 \%$. It appears that having a cell phone and having a land line are independent, since the probabilities are roughly the same.

32. Organize the percentages in a Venn diagram.

a) $13.7 \%$ of the respondents were under 30 and did not snore.
b) According to this survey, snoring is not independent of age. $36.8 \%$ of the 995 adults snored, but
$32 /(32+49.5)=39.3 \%$ of those over 30 snored.
33. According to the poll, party affiliation is not independent of gender. Overall, $(36+48) / 202=41.6 \%$ of the respondents were Democrats. Of the men, only $36 / 105=$ $34.3 \%$ were Democrats.
34. According to the survey, country of origin of the car is not independent of type of driver. $(33+12) / 359=12.5 \%$ of the cars were of European origin, but about 33/195 = $16.9 \%$ of the students drive European cars.
35. Organize using a tree diagram.

a) No, the flight leaving on time and the luggage making the connection are not independent events. The probability that the luggage makes the connection is dependent on whether or not the flight is on time. The probability is 0.95 if the flight is on time, and only 0.65 if it is not on time.
b) $P($ Luggage $)=P($ On time $\cap$ Luggage $)+P($ Not on time $\cap$ Luggage $)=(0.15)(0.95)+(0.85)(0.65)=0.695$
36. a) Yes, there is evidence to suggest that a freshman's chances to graduate depend upon what kind of high school the student attended. The graduation rate for public school students is $75 \%$, while the graduation rate for others is $90 \%$. If the high school attended was independent of college graduation, these percentages would be the same.

b) $P($ Graduate $)=P($ Public $\cap$ Graduate $)+P($ Not public $\cap$ Graduate $)=(0.7)(0.75)+(0.3)(0.9)=0.79$
Overall, $79.5 \%$ of freshmen are expected to eventually graduate.
37. Refer to the tree diagram constructed for Exercise 35.
$\frac{(0.85)(0.35)}{(0.15)(0.05)+(0.85)(0.35)}=0.975$
If you pick Leah up at the Denver airport and her luggage is not there, the probability that her first flight was delayed is 0.975 .
38. Refer to the tree diagram constructed for Exercise 36.
$\frac{(0.7)(0.75)}{(0.7)(0.75)+(0.3)(0.9)}=0.660$
Overall, $66.0 \%$ of the graduates of the private college went to public high schools.
39. Organize the information in a tree diagram.

a) No, absenteeism is not independent of shift worked. The rate of absenteeism for the night shift is $2 \%$, while the rate for the day shift is only $1 \%$. If the two were independent, the percentages would be the same.
b) $P($ Absent $)=P($ Day $\cap$ Absent $)+P($ Night $\cap$

Absent $)=(0.6)(0.01)+(0.4)(0.02)=0.014$
The overall rate of absenteeism at this company is $1.4 \%$.
40. Organize the information into a tree diagram.

a) The lung condition and smoking are not independent, since rates of the lung condition are different for smokers and nonsmokers. $57 \%$ of smokers have the lung condition by age 60 , while only $13 \%$ of nonsmokers have the condition by age 60 .
b) P (Smoker $\cap$ Lung Conditon) $+\mathrm{P}($ Nonsmoker $\cap$ Lung Condition $)=(0.23)(0.57)+(0.77)(0.13) \approx 0.231$
The probability that a randomly selected 60 -year-old has the lung condition is about 0.231 .
41. Refer to the tree diagram constructed for Exercise 39.
$\frac{(0.4)(0.02)}{(0.6)(0.01)+(0.4)(0.02)}=0.571$
Approximately $57.1 \%$ of the company's absenteeism occurs on the night shift.
42. Refer to the tree diagram constructed for Exercise 40.
$\frac{(0.23)(0.57)}{(0.23)(0.57)+(0.77)(0.13)}=0.567$
The probability that someone who has the lung condition by age 60 is a smoker is approximately $56.7 \%$.
43. Organize the information into a tree diagram.

a) $P($ Detain $\mid$ Not Drinking $)=0.2$
b) $(0.12)(0.8)+(0.88)(0.2)=0.272$
c) $\frac{(0.12)(0.8)}{(0.12)(0.8)+(0.88)(0.2)}=0.353$
44. Organize the information in a tree diagram.


The probability that a job applicant rejected under suspicion of dishonesty is actually trustworthy is about 0.814 .
45. Organize the information in a tree diagram.


If you hear a dish break, the probability that Chuck is on the job is approximately 0.563 .

