

- Roulette. A casino claims that its roulette wheel is truly random. What should that claim mean?
- Rain. The weather reporter on TV makes predictions such as a 25% chance of rain. What do you think is the meaning of such a phrase?
- Winter. Comment on the following quotation:  
*"What I think is our best determination is it will be a colder than normal winter," said Pamela Naber Knox, a Wisconsin state climatologist. "I'm basing that on a couple of different things. First, in looking at the past few winters, there has been a lack of really cold weather. Even though we are not supposed to use the law of averages, we are due."*
- Snow. After an unusually dry autumn, a radio announcer is heard to say, "Watch out! We'll pay for these sunny days later on this winter." Explain what he's trying to say, and comment on the validity of his reasoning.
- Cold streak. A batter who had failed to get a hit in seven consecutive times at bat then hits a game-winning home run. When talking to reporters afterward, he says he was very confident that last time at bat because he knew he was "due for a hit." Comment on his reasoning.
- Crash. Commercial airplanes have an excellent safety record. Nevertheless, there are crashes occasionally, with the loss of many lives. In the weeks following a crash, airlines often report a drop in the number of passengers, probably because people are afraid to risk flying.
  - A travel agent suggests that, since the law of averages makes it highly unlikely to have two plane crashes within a few weeks of each other, flying soon after a crash is the safest time. What do you think?
  - If the airline industry proudly announces that it has set a new record for the longest period of safe flights, would you be reluctant to fly? Are the airlines due to have a crash?
- Fire insurance. Insurance companies collect annual payments from homeowners in exchange for paying to rebuild houses that burn down.
  - Why should you be reluctant to accept a \$300 payment from your neighbor to replace his house should it burn down during the coming year?
  - Why can the insurance company make that offer?
- Jackpot. On January 20, 2000, the International Gaming Technology company issued a press release: *(LAS VEGAS, Nev.)-Cynthia Jay was smiling ear to ear as she walked into the news conference at The Desert Inn Resort in Las Vegas today, and well she should. Last night, the 37-year-old cocktail waitress won the world's largest slot jackpot-\$34,959,458---on a Megabucks machine. She said she had played \$27 in the machine when the jackpot hit. Nevada Megabucks has produced 49 major winners in its 14-year history. The top jackpot builds from a base amount of \$7 million and can be won with a 3-coin (\$3) bet.*
  - How can the Desert Inn afford to give away millions of dollars on a \$3 bet?
  - Why did the company issue a press release? Wouldn't most businesses want to keep such a huge loss quiet?
- Spinner. The plastic arrow on a spinner for a child's game stops rotating to point at a color that will determine what happens next. Are the following probability assignments possible? Why or why not?
 

Probabilities of ...				
	Red	Yellow	Green	Blue
a)	0.25	0.25	0.25	0.25
b)	0.10	0.20	0.30	0.40
c)	0.20	0.30	0.40	0.50
d)	0	0	1.00	0
e)	0.10	0.20	1.20	-1.50
- Scratch off. Many stores run "secret sales": Shoppers receive cards that determine how large a discount they get, but the percentage is revealed by scratching off that black stuff (what *is* that?) only after the purchase has been totaled at the cash register. The store is required to reveal (in the fine print) the distribution of discounts available. Are these probability assignments plausible? Why or why not?
 

Probabilities of ...				
	10% off	20% off	30% off	50% off
a)	0.20	0.20	0.20	0.20
b)	0.50	0.30	0.20	0.10
c)	0.80	0.10	0.05	0.05
d)	0.75	0.25	0.25	-0.25
e)	1.00	0	0	0
- Car repairs. A consumer organization estimates that over a 1-year period 17% of cars will need to be repaired once, 7% will need repairs twice, and 4% will require three or more repairs. What is the probability that a car chosen at random will need
  - no repairs?
  - no more than one repair?
  - some repairs?

12. Stats projects. In a large Introductory Statistics lecture hall, the professor reports that 55% of the students enrolled have never taken a Calculus course, 32% have taken only one semester of Calculus, and the rest have taken two or more semesters of Calculus. The professor randomly assigns students to groups of three to work on a project for the course. What is the probability that the first groupmate you meet has studied
- two or more semesters of Calculus?
  - some Calculus?
  - no more than one semester of Calculus?
13. More repairs. Consider again the auto repair rates described in Exercise 11. If you own two cars, what is the probability that
- neither will need repair?
  - both will need repair?
  - at least one car will need repair?
14. Another project. You are assigned to be part of a group of three students from the Intra Stats class described in Exercise 12. What is the probability that, of your other two groupmates,
- neither has studied Calculus?
  - both have studied at least one semester of Calculus?
  - at least one has had more than one semester of Calculus?
15. Repairs, again. You used the Multiplication Rule to calculate repair probabilities for your cars in Exercise 13.
- What must be true about your cars in order to make that approach valid?
  - Do you think this assumption is reasonable? Explain.
16. Final project. You used the Multiplication Rule to calculate probabilities about the Calculus background of your Statistics groupmates in Exercise 14.
- What must be true about the groups in order to make that approach valid?
  - Do you think this assumption is reasonable? Explain.
17. Energy. A Gallup poll in March 2001 asked 1005 U.S. adults how the United States should deal with the current energy situation: by more production, more conservation, or both? Here are the results:

Response	Number
More production	332
More conservation	563
Both	80
No opinion	30
Total	1005

- If we select a person at random from this sample of 1005 adults,
- what is the probability that the person responded "More production"?
  - what is the probability that the person responded "Both" or had no opinion?

18. All about Bill. A Gallup Poll in June 2004 asked 1005 U.S. adults how likely they were to read Bill Clinton's autobiography *My Life*. Here's how they responded:

Response	Number
Will definitely read it	90
Will probably read it	211
Will probably not read it	322
Will definitely not read it	382
Total	1005

- If we select a person at random from this sample of 1005 adults,
- what is the probability that the person responded "Will definitely not read it"?
  - what is the probability that the person will probably or definitely read it?
19. More energy. Exercise 17 shows the results of a Gallup Poll about energy. Suppose we select three people at random from this sample.
- What is the probability that all three responded "More conservation"?
  - What is the probability that none responded "Both"?
  - What assumption did you make in computing these probabilities?
  - Explain why you think that assumption is reasonable.
20. More about Bill. Consider the results of the poll about President Clinton's book, summarized in Exercise 18. Let's call someone who responded that they would definitely or probably read it a "likely reader" and the other two categories, "unlikely reader." If we select two people at random from this sample,
- what is the probability that both are likely readers?
  - what is the probability that neither is a likely reader?
  - what is the probability that one is a likely reader and one isn't?
  - What assumption did you make in computing these probabilities?
  - Explain why you think that assumption is reasonable.

21. Polling. As mentioned in the chapter, opinion-polling organizations contact their respondents by sampling random telephone numbers. Although interviewers now can reach about 76% of U.S. households, the percentage of those contacted who agree to cooperate with the survey has fallen from 58% in 1997 to only 38% in 2003 (Pew Research Center for the People and the Press). Each household, of course, is independent of the others.
- What is the probability that the next household on the list will be contacted, but will refuse to cooperate?
  - What is the probability (in 2003) of failing to contact a household or of contacting the household but not getting them to agree to the interview?
  - Show another way to calculate the probability in part b.
22. Polling, part II. According to Pew Research, the contact rate (probability of contacting a selected household) in 1997 was 69% and in 2003 was 76%. However, the cooperation rate (probability of someone at the contacted household agreeing to be interviewed) was 58% in 1997 and dropped to 38% in 2003.
- What is the probability (in 2003) of obtaining an interview with the next household on the sample list? (To obtain an interview, an interviewer must both contact the household and then get agreement for the interview.)
  - Was it more likely to obtain an interview from a randomly selected household in 1997 or in 2003?
23. M&M's. The Masterfoods company says that before the introduction of purple, yellow candies made up 20% of their plain M&M's, red another 20%, and orange, blue, and green each made up 10%. The rest were brown.
- If you pick an M&M at random, what is the probability that
    - it is brown?
    - it is yellow or orange?
    - it is not green?
    - it is striped?
  - If you pick three M&M's in a row, what is the probability that
    - they are all brown?
    - the third one is the first one that's red?
    - none are yellow?
    - at least one is green?
24. Blood. The American Red Cross says that about 45% of the U.S. population has Type O blood, 40% Type A, 11% Type B, and the rest Type AB.
- Someone volunteers to give blood. What is the probability that this donor
    - has Type AB blood?
    - has Type A or Type B?
    - is not Type O?
  - Among four potential donors, what is the probability that
    - all are Type O?
    - no one is Type AB?
    - they are not all Type A?
    - at least one person is Type B?
25. Disjoint or independent? In Exercise 23 you calculated probabilities of getting various M&M's. Some of your answers depended on the assumption that the outcomes described were *disjoint*; that is, they could not both happen at the same time. Other answers depended on the assumption that the events were *independent*; that is, the occurrence of one of them doesn't affect the probability of the other. Do you understand the difference between disjoint and independent?
- If you draw one M&M, are the events of getting a red one and getting an orange one disjoint or independent or neither?
  - If you draw two M&M's one after the other, are the events of getting a red on the first and a red on the second disjoint or independent or neither?
  - Can disjoint events ever be independent? Explain.
26. Disjoint or independent? In Exercise 24 you calculated probabilities involving various blood types. Some of your answers depended on the assumption that the outcomes described were *disjoint*; that is, they could not both happen at the same time. Other answers depended on the assumption that the events were *independent*; that is, the occurrence of one of them doesn't affect the probability of the other. Do you understand the difference between disjoint and independent?
- If you examine one person, are the events that the person is Type A and that the person is Type B disjoint or independent or neither?
  - If you examine two people, are the events that the first is Type A and the second Type B disjoint or independent or neither?
  - Can disjoint events ever be independent? Explain.

27. Dice. You roll a fair die three times. What is the probability that
- you roll all 6's?
  - you roll all odd numbers?
  - none of your rolls gets a number divisible by 3?
  - you roll at least one 5?
  - the numbers you roll are not all 5's?
28. Slot machine. A slot machine has three wheels that spin independently. Each has 10 equally likely symbols: 4 bars, 3 lemons, 2 cherries, and a bell. If you play, what is the probability
- you get 3 lemons?
  - you get no fruit symbols?
  - you get 3 bells (the jackpot)?
  - you get no bells?
  - you get at least one bar (an automatic loser)?
29. Champion bowler. A certain bowler can bowl a strike 70% of the time. What is the probability that she
- goes three consecutive frames without a strike?
  - makes her first strike in the third frame?
  - has at least one strike in the first three frames?
  - bowls a perfect game (12 consecutive strikes)?
30. The train. To get to work, a commuter must cross train tracks. The time the train arrives varies slightly from day to day, but the commuter estimates he'll get stopped on about 15% of work days. During a certain 5-day work week, what is the probability that he
- gets stopped on Monday and again on Tuesday?
  - gets stopped for the first time on Thursday?
  - gets stopped every day?
  - gets stopped at least once during the week?
31. Voters. Suppose that in your city 37% of the voters are registered as Democrats, 29% as Republicans, and 11% as members of other parties (Liberal, Right to Life, Green, etc.). Voters not aligned with any official party are termed "Independent." You are conducting a poll by calling registered voters at random. In your first three calls, what is the probability you talk to
- all Republicans?
  - no Democrats?
  - at least one Independent?
32. Religion. Census reports for a city indicate that 62% of residents classify themselves as Christian, 12% as Jewish, and 16% as members of other religions (Muslims, Buddhists, etc.). The remaining residents classify themselves as nonreligious. A polling organization seeking information about public opinions wants to be sure to talk with people holding a variety of religious views, and makes random phone calls. Among the first four people they call, what is the probability they reach
- all Christians?
  - no Jews?
  - at least one person who is nonreligious?
33. Tires. You bought a new set of four tires from a manufacturer who just announced a recall because 2% of those tires are defective. What is the probability that at least one of yours is defective?
34. Pepsi. For a sales promotion, the manufacturer places winning symbols under the caps of 10% of all Pepsi bottles. You buy a six-pack. What is the probability that you win something?
35. 9/11? On September 11, 2002, the first anniversary of the terrorist attack on the World Trade Center, the New York State Lottery's daily number came up 9-1-1. An interesting coincidence or a cosmic sign?
- What is the probability that the winning three numbers match the date on any given day?
  - What is the probability that a whole year passes without this happening?
  - What is the probability that the date and winning lottery number match at least once during any year?

## Answers

1. If a roulette wheel is to be considered truly random, then each outcome is equally likely to occur, and knowing one outcome will not affect the probability of the next. Additionally, there is an implication that the outcome is not determined through the use of an electronic random number generator.
2. When a weather forecaster makes a prediction such as a 25% chance of rain, this means that when weather conditions are like they are now, rain happens 25% of the time in the long run.
3. Although acknowledging that there is no law of averages, Knox attempts to use the law of averages to predict the severity of the winter. Some winters are harsh and some are mild over the long run, and knowledge of this can help us to develop a long-term probability of having a harsh winter. However, probability does not compensate for odd occurrences in the short term. Suppose that the probability of having a harsh winter is 30%. Even if there are several mild winters in a row, the probability of having a harsh winter is still 30%.
4. The radio announcer is referring to the “law of averages”, which is not true. Probability does not compensate for deviations from the expected outcome in the recent past. The weather is not more likely to be bad later on in the winter because of a few sunny days in autumn. The weather makes no conscious effort to even things out, which is what the announcer’s statement implies.
5. There is no such thing as being “due for a hit”. This statement is based on the so-called law of averages, which is a mistaken belief that probability will compensate in the short term for odd occurrences in the past. The batter’s chance for a hit does not change based on recent successes or failures.
6. a) There is no such thing as the “law of averages”. The overall probability of an airplane crash does not change due to recent crashes.  
b) Again, there is no such thing as the “law of averages”. The overall probability of an airplane crash does not change due to a period in which there were no crashes. It makes no sense to say a crash is “due”. If you say this, you are expecting probability to compensate for strange events in the past.
7. a) It would be foolish to insure your neighbor’s house for \$300. Although you would probably simply collect \$300, there is a chance you could end up paying much more than \$300. That risk probably is not worth the \$300.  
b) The insurance company insures many people. The overwhelming majority of customers pay the insurance and never have a claim. The few customers who do have a claim are offset by the many who simply send their premiums without a claim. The relative risk to the insurance company is low.
8. a) The Desert Inn can afford to give away millions of dollars on a \$3 bet because almost all of the people who bet do not win the jackpot.  
b) The press release generates publicity, which entices more people to come and gamble. Of course, the casino wants people to play, because the overall odds are always in favor of the casino. The more people who gamble, the more the casino makes in the long run. Even if that particular slot machine has paid out more than it ever took in, the publicity it gives to the casino more than makes up for it.
9. a) This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1.  
b) This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1.  
c) This is not a legitimate probability assignment. Although each outcome has probability between 0 and 1, inclusive, the sum of the probabilities is greater than 1.  
d) This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1. However, this game is not very exciting!  
e) This probability assignment is not legitimate. The sum of the probabilities is 0, and there is one probability,  $-1.5$ , that is not between 0 and 1, inclusive.
10. a) This is not a legitimate assignment. Although each outcome has probability between 0 and 1, inclusive, the sum of the probabilities is less than 1.  
b) This is not a legitimate probability assignment. Although each outcome has probability between 0 and 1, inclusive, the sum of the probabilities is greater than 1.  
c) This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive, and the sum of the probabilities is 1.  
d) This probability assignment is not legitimate. Although the sum of the probabilities is 1, there is one probability,  $-0.25$ , that is not between 0 and 1, inclusive.  
e) This is a legitimate probability assignment. Each outcome has probability between 0 and 1, inclusive,

and the sum of the probabilities is 1. This is also known as a 10% off sale!

11. Since all of the events listed are disjoint, the addition rule can be used.

a)  $P(\text{no repairs}) = 1 - P(\text{some repairs}) = 1 - (0.17 + 0.07 + 0.04) = 1 - (0.28) = 0.72$

b)  $P(\text{no more than one repair}) = P(\text{no repairs} \cup \text{one repair}) = 0.72 + 0.17 = 0.89$

c)  $P(\text{some repairs}) = P(\text{one} \cup \text{two} \cup \text{three} \cup \text{more repairs}) = 0.17 + 0.07 + 0.04 = 0.28$

12. Since all of the events listed are disjoint, the addition rule can be used.

a)  $P(\text{two or more semesters of Calculus}) = 1 - (0.55 + 0.32) = 0.13$

b)  $P(\text{some Calculus}) = P(\text{one semester} \cup \text{two or more semesters}) = 0.32 + 0.13 = 0.45$

c)  $P(\text{no more than one semester}) = P(\text{no Calculus} \cup \text{one semester}) = 0.55 + 0.32 = 0.87$

13. Assuming that repairs on the two cars are independent from one another, the multiplication rule can be used. Use the probabilities of events from Exercise 11 in the calculations.

a)  $P(\text{neither will need repair}) = (0.72)(0.72) = 0.5184$

b)  $P(\text{both will need repair}) = (0.28)(0.28) = 0.0784$

c)  $P(\text{at least one will need repair}) = 1 - P(\text{neither will need repair}) = 1 - (0.72)(0.72) = 0.4816$

14. Since students with Calculus backgrounds are independent from one another, use the multiplication rule. Use the probabilities of events from Exercise 12 in the calculations.

a)  $P(\text{neither has studied Calculus}) = (0.55)(0.55) = 0.3025$

b)  $P(\text{both have studied at least one semester of Calculus}) = (0.45)(0.45) = 0.2025$

c)  $P(\text{at least one has had more than one semester of Calculus}) = 1 - P(\text{neither has studied more than one semester of Calculus}) = 1 - (0.87)(0.87) = 0.2431$

15. a) The repair needs for the two cars must be independent of one another.

b) This may not be reasonable. An owner may treat the two cars similarly, taking good (or poor) care of both. This may decrease (or increase) the likelihood that each needs to be repaired.

16. a) The Calculus backgrounds of the students must be independent of one another.

b) Since the professor assigned the groups at random, the Calculus backgrounds are independent.

17. a)  $P(\text{response is "More production"}) = 332/1005 = 0.330$

b)  $P(\text{response is "Both"} \cup \text{"No opinion"}) = (80 + 30)/1005 = 0.109$

18. a)  $P(\text{response is "Will definitely not read it"}) = 382/1005 = 0.380$

b)  $P(\text{response is "Will probably"} \cup \text{"Will definitely read it"}) = (90+211)/1005 = 0.300$

19. More energy.

a)  $P(\text{all three respond "More conservation"}) = \left(\frac{563}{1005}\right)\left(\frac{563}{1005}\right)\left(\frac{563}{1005}\right) = 0.176$

b)  $P(\text{none respond "Both"}) =$

$\left(\frac{925}{1005}\right)\left(\frac{925}{1005}\right)\left(\frac{925}{1005}\right) = 0.780$

c) In order to compute the probabilities, we must assume that responses are independent.

d) It is reasonable to assume that responses are independent, since the three people were chosen at random.

20. a)  $P(\text{both are likely readers}) =$

$\left(\frac{301}{1005}\right)\left(\frac{301}{1005}\right) = 0.090$

b)  $P(\text{neither is a likely reader}) =$

$\left(\frac{704}{1005}\right)\left(\frac{704}{1005}\right) = 0.491$

c)  $P(\text{one is a likely reader} \cap \text{the other is not}) =$

$\left(\frac{301}{1005}\right)\left(\frac{704}{1005}\right) + \left(\frac{704}{1005}\right)\left(\frac{301}{1005}\right) = 0.420$

d) In order to compute the probabilities, we must assume that responses are independent.

e) It is reasonable to assume that responses are independent, since the two people were chosen at random.

21. a)  $P(\text{household is contacted} \cap \text{household refuses to cooperate})$

$= P(\text{household is contacted})P(\text{household refuses} | \text{contacted}) = (0.76)(1 - 0.38) = 0.4712$

b)  $P(\text{failing to contact household} \cup \text{contacting} \cap \text{not getting the interview})$

$= P(\text{failing to contact}) + P(\text{contacting household} | \text{not getting the interview} | \text{contacted})$

$= (1 - 0.76) + (0.76)(1 - 0.38) = 0.7112$

c) The question in part b covers all possible occurrences except contacting the house and getting the interview. The probability could also be calculated by subtracting the probability of the complement of the event from 22a.

$P(\text{failing to contact household} \cup \text{contacting} \cap \text{not getting the interview}) = 1 - P(\text{contacting the household} \cap \text{getting the interview}) = 1 - (0.76)(0.38) = 0.7112$

22. a)  $P(2003 \text{ household is contacted} \cap \text{household cooperates}) = P(\text{household is contacted})P(\text{household cooperates} | \text{contacted}) = (0.76)(0.38) = 0.2888$

b)  $P(1997 \text{ household is contacted} \cap \text{cooperates}) = P(\text{household is contacted})P(\text{household cooperates} | \text{contacted}) = (0.69)(0.58) = 0.4002$

It was more likely for pollsters to obtain an interview at the next household in 1997 than in 2003.

23. a) Since all of the events are disjoint (an M&M can't be two colors at once!), use the addition rule where applicable.

1.  $P(\text{brown}) = 1 - P(\text{not brown}) = 1 - P(\text{yellow} \cup \text{red} \cup \text{orange} \cup \text{blue} \cup \text{green})$

$= 1 - (0.20 + 0.20 + 0.10 + 0.10 + 0.10) = 0.30$

2.  $P(\text{yellow} \cup \text{orange}) = 0.20 + 0.10 = 0.30$

3.  $P(\text{not green}) = 1 - P(\text{green}) = 1 - 0.10 = 0.90$

4.  $P(\text{striped}) = 0$

b) Since the events are independent (picking out one M&M doesn't affect the outcome of the next pick), the multiplication rule may be used.

1.  $P(\text{all three are brown}) = (0.30)(0.30)(0.30) = 0.027$

2.  $P(\text{the third one is the first one that is red}) = P(\text{not red} \cap \text{not red} \cap \text{red}) = (0.80)(0.80)(0.20) = 0.128$

3.  $P(\text{no yellow}) = P(\text{not yellow} \cap \text{not yellow} \cap \text{not yellow}) = (0.80)(0.80)(0.80) = 0.512$

4.  $P(\text{at least one is green}) = 1 - P(\text{none are green}) = 1 - (0.90)(0.90)(0.90) = 0.271$

24. a) Since all of the events are disjoint (a person cannot have more than one blood type!), use the addition rule where applicable.

1.  $P(\text{Type AB}) = 1 - P(\text{not Type AB}) = 1 - P(\text{Type O} \cup \text{Type A} \cup \text{Type B}) = 1 - (0.45 + 0.40 + 0.11) = 0.04$

2.  $P(\text{Type A} \cup \text{Type B}) = 0.40 + 0.11 = 0.51$

3.  $P(\text{not Type O}) = 1 - P(\text{Type O}) = 1 - 0.45 = 0.55$

b) Since the events are independent (one person's blood type doesn't affect the blood type of the next), the multiplication rule may be used.

1.  $P(\text{all four are Type O}) = (0.45)(0.45)(0.45)(0.45) \approx 0.041$

2.  $P(\text{no one is Type AB}) = P(\text{not AB} \cap \text{not AB} \cap \text{not AB} \cap \text{not AB}) = (0.96)(0.96)(0.96)(0.96) \approx 0.849$

3.  $P(\text{they are not all Type A}) = 1 - P(\text{all Type A}) = 1 - (0.40)(0.40)(0.40)(0.40) = 0.9744$

4.  $P(\text{at least one person is Type B}) = 1 - P(\text{no one is Type B}) = 1 - (0.89)(0.89)(0.89)(0.89) \approx 0.373$

25. a) For one draw, the events of getting a red M&M and getting an orange M&M are disjoint events.

Your single draw cannot be both red and orange.

b) For two draws, the events of getting a red M&M on the first draw and a red M&M on the second draw are independent events. Knowing that the first draw is red does not influence the probability of getting a red M&M on the second draw.

c) Disjoint events can never be independent. Once you know that one of a pair of disjoint events has occurred, the other one cannot occur, so its probability has become zero. For example, consider drawing one M&M. If it is red, it cannot possibly be orange. Knowing that the M&M is red influences the probability that the M&M is orange. It's zero. The events are not independent.

26. a) For one person, the events of having Type A blood and having Type B blood are disjoint events. One person cannot have both Type A and Type B blood.

b) For two people, the events of the first having Type A blood and the second having Type B blood are independent events. Knowing that the first person has Type A blood does not influence the probability of the second person having Type B blood.

c) Disjoint events can never be independent. Once you know that one of a pair of disjoint events has occurred, the other one cannot occur, so its probability has become zero. For example, consider selecting one person, and checking his or her blood type. If the person's blood type is Type A, it cannot possibly be Type B. Knowing that the person's blood type is Type A influences the probability that the person's blood type is Type B. It's zero. The events are not independent.

27. a)  $P(6) = \frac{1}{6}$ , so  $P(\text{all 6's}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = 0.005$

b)  $P(\text{odd}) = P(1 \cup 3 \cup 5) = \frac{3}{6}$ , so  $P(\text{all odd}) =$

$\left(\frac{3}{6}\right)\left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = 0.125$

c)  $P(\text{not divisible by 3}) = P(1 \cup 2 \cup 4 \cup 5) = \frac{4}{6}$ ,

so  $P(\text{none divisible by 3}) = \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{4}{6}\right) = 0.296$

d)  $P(\text{at least one 5}) = 1 - P(\text{no 5's}) =$

$1 - \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) = 0.421$

e)  $P(\text{not all 5's}) = 1 - P(\text{all 5's}) =$

$$1 - \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = 0.995$$

28. Each wheel runs independently of the others, so the multiplication rule may be used.

a)  $P(\text{lemon on 1 wheel}) = 0.30$ , so  $P(3 \text{ lemons}) = (0.30)(0.30)(0.30) = 0.027$

b)  $P(\text{bar} \cup \text{bell on 1 wheel}) = 0.50$ , so  $P(\text{no fruit symbols}) = (0.50)(0.50)(0.50) = 0.125$

c)  $P(\text{bell on 1 wheel}) = 0.10$ , so  $P(3 \text{ bells}) = (0.10)(0.10)(0.10) = 0.001$

d)  $P(\text{no bell on 1 wheel}) = 0.90$ , so  $P(\text{no bells on 3 wheels}) = (0.90)(0.90)(0.90) = 0.729$

e)  $P(\text{no bar on 1 wheel}) = 0.60$ .

$$P(\text{at least one bar on 3 wheels}) = 1 - P(\text{no bars}) = 1 - (0.60)(0.60)(0.60) = 0.784$$

29. Assuming each frame is independent of others, so the multiplication rule may be used.

a)  $P(\text{no strikes in 3 frames}) = (0.30)(0.30)(0.30) = 0.027$

b)  $P(\text{makes first strike in the third frame}) = (0.30)(0.30)(0.70) = 0.063$

c)  $P(\text{at least one strike in the first three frames}) = 1 - P(\text{no strikes}) = 1 - (0.30)^3 = 0.973$

d)  $P(\text{perfect game}) = (0.70)^{12} = 0.014$

30. Assuming the arrival time is independent from one day to the next, the multiplication rule may be used.

a)  $P(\text{gets stopped Monday} \cap \text{gets stopped Tuesday}) = (0.15)(0.15) = 0.0225$

b)  $P(\text{gets stopped for the first time on Thursday}) = (0.85)(0.85)(0.85)(0.15) \approx 0.092$

c)  $P(\text{gets stopped every day}) = (0.15)^5 \approx 0.00008$

d)  $P(\text{gets stopped at least once}) = 1 - P(\text{never gets stopped}) = 1 - (0.85)^5 \approx 0.556$

31. Since you are calling at random, one person's political affiliation is independent of another's. The multiplication rule may be used.

a)  $P(\text{all Republicans}) = (0.29)(0.29)(0.29) \approx 0.024$

b)  $P(\text{no Democrats}) = (1 - 0.37)(1 - 0.37)(1 - 0.37) \approx 0.25$

c)  $P(\text{at least one Independent}) = 1 - P(\text{no Independents}) = 1 - (0.77)(0.77)(0.77) \approx 0.543$

32. Since you are calling at random, one person's religion is independent of another's. The multiplication rule may be used.

a)  $P(\text{all Christian}) = (0.62)(0.62)(0.62)(0.62) \approx 0.148$

b)  $P(\text{no Jews}) = (1 - 0.12)(1 - 0.12)(1 - 0.12)(1 - 0.12) \approx 0.600$

c)  $P(\text{at least one person who is nonreligious}) = 1 -$

$$P(\text{no nonreligious people})$$

$$= 1 - (0.90)(0.90)(0.90)(0.90) = 0.3439$$

33. Assume that the defective tires are distributed randomly to all tire distributors so that the events can be considered independent. The multiplication rule may be used.

$$P(\text{at least one of four tires is defective}) = 1 - P(\text{none are defective})$$

$$= 1 - (0.98)(0.98)(0.98)(0.98) \approx 0.078$$

34. Assume that the winning caps are distributed randomly, so that the events can be considered independent. The multiplication rule may be used.

$$P(\text{you win something}) = 1 - P(\text{you win nothing}) = 1 - (0.90)^6 = 0.469$$

35. a) For any date with a valid three-digit date, the chance is 0.001, or 1 in 1000. For many dates in October through December, the probability is 0. For example, there is no way three digits will make 1015, to match October 15.

b) There are 65 days when the chance to match is 0. (October 10 through October 31, November 10 through November 30, and December 10 through December 31.) That leaves 300 days in a year (that is not a leap year) in which a match might occur.

$$P(\text{no matches in 300 days}) = (0.999)^{300} = 0.741$$

c)  $P(\text{at least one match in a year}) = 1 - P(\text{no matches in a year}) = 1 - 0.741 \approx 0.259$

d)  $P(\text{at least one match on 9/11 in one of the 50 states})$

$$= 1 - P(\text{no matches in 50 states}) =$$

$$1 - (0.999)^{50} = 0.049$$

36. a) Your thinking is correct. There are 42 cards left in the deck, 26 black and only 16 red.

b) This is not an example of the Law of Large Numbers. There is no "long run". You'll see the entire deck after 52 cards, and you know there will be 26 of each color then.